CHAPTER 9

MORPHOLOGICAL IMAGE PROCESSING

CHAPTER 9: MORPHOLOGICAL IMAGE PROCESSING

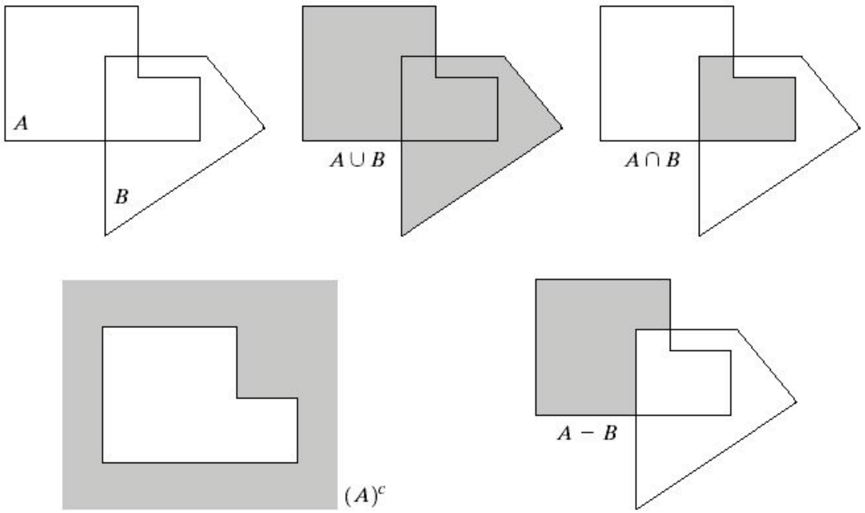
- **Morphology**: a branch of biology that deals with the form and structure of animals and plants.
- **Mathematical morphology**: extraction of image components that are useful in the representation and description of regional shape.
  - Boundaries
  - Skeletons
  - Convex hull
- The language of mathematical morphology is **set theory**.
- **Sets** in mathematical morphology represent objects in an image.
- In binary images, the sets are **members** of Z^2.
- Until this chapter, the I/O of DIP methods were images.
- Now, we have a transition to DIP methods where I are images but O are attributes extracted from those images.

$\hat{B}$

BASIC CONCEPTS FROM SET THEORY

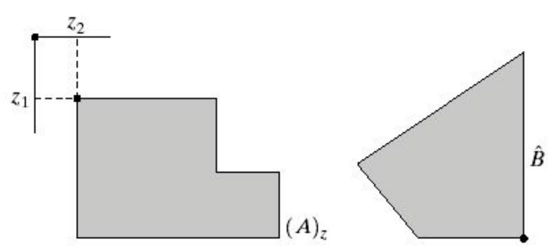
- $A \in Z^2$.
- $a = (a_1, a_2) \in A$.
- Empty set: the set with <u>no</u> elements.
- $w = (w_1,w_2)$ & $C = \{w|w = -d,\ d \in D\}$: C is the set of elements w s.t. w is formed by multiplying each of the 2 coordinates of all the elements of D by -1.
- $A \subseteq B$: A is a subset of B.
- $C = A \cup B$: C is the union of A and B.
- $D = A \cap B$: D is the intersection of A and B.
- $A \cap B = \varnothing$: A and B are disjoint sets.
- $A^c = \{w|w \notin A\}$: The complement of A.
- $A - B = \{w|w \in A,\ w \notin B\} = A \cap B^c$: The difference of A and B.
- $\hat{B} = \{w|w = -b,\ b \in B\}$: Reflection of B.
- $(A)_z = \{c|c = A + z,\ a \in A\}$: Translation of A by $z = (z_1,z_2)$

BASIC SET OPERATIONS



a b c
d e

FIGURE 9.1
(a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.



a b

FIGURE 9.2
(a) Translation of A by z.
(b) Reflection of B. The sets A and B are from Fig. 9.1.

3 BASIC LOGICAL OPERATIONS

TABLE 9.1
The three basic logical operations.

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

Other logical operations can be constructed using the above definitions.

<u>Example:</u>

p	q	p XOR q
0	0	0
0	1	1
1	0	1
1	1	0

LOGICAL OPERATIONS BETWEEN BINARY IMAGES

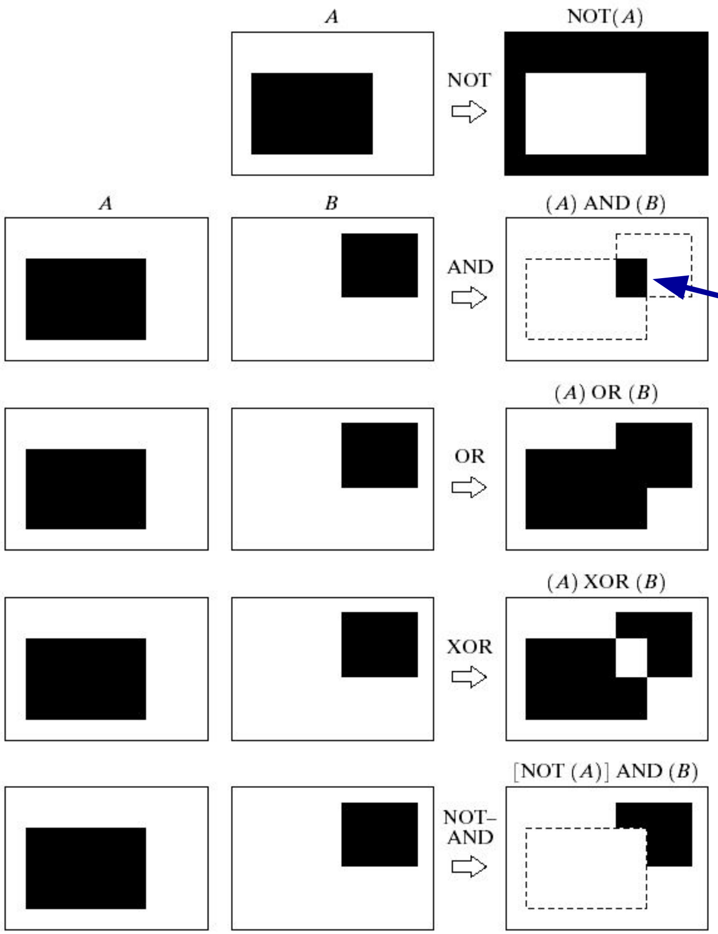


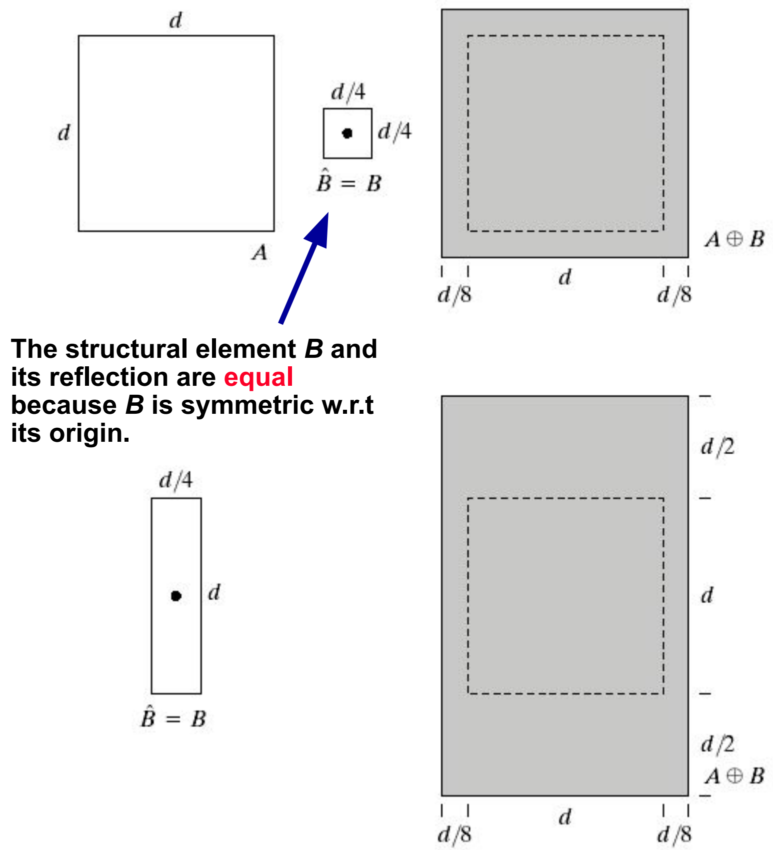
FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

p	q	*p* AND *q*
0	0	0
0	1	0
1	0	0
1	1	1

DILATION

- Dilation and erosion: Operations fundamental to morphological processing.
- Dilation <u>expands</u> an image, and <u>erosion</u> shrinks it.
- Many morphological algorithms in this chapter are based on these 2 primitives.
- $A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$: dilation of *A* and *B*
  - Obtain the reflection of *B* about its origin, and shift this reflection by *z*.
  - The set of all displacements *z* s.t. $\hat{B}$ and A overlap by at least one element.
  - *B*: the structural element
- Alternative definitions of dilation
  - $$A \oplus B = \{w \in Z^2 \mid w = a + b, a \in A, b \in B\}$$
  - $$A \oplus B = \bigcup_{b \in B} (A)_b$$

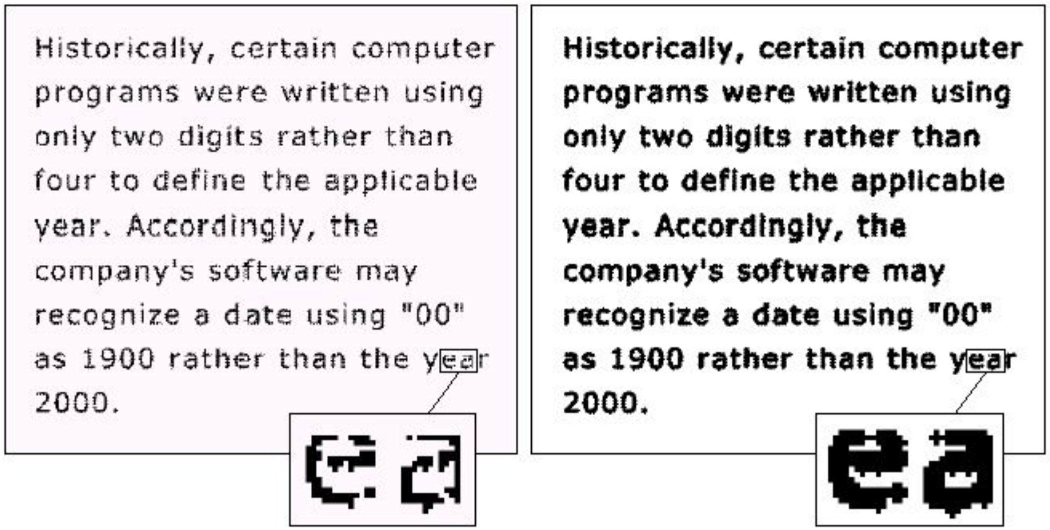
2 EXAMPLES OF DILATION



a	b	c
d		e

FIGURE 9.4
(a) Set A.
(b) Square structuring element (dot is the center).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element.
(e) Dilation of A using this element.

AN APPLICATION OF DILATION: BRIDGING GAPS



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

EROSION

- $A \ominus B = \{z \mid (B)_z \subseteq A\}$: Erosion of ***A*** by ***B***.
  - The set of all points ***z*** s.t. ***B***, translated by ***z***, is contained in ***A***.
- **Alternative definitions of erosion**
  - $A \ominus B = \{w \in Z^2 \mid w + b \in A, \forall b \in B\}$
  - $A \ominus B = \bigcap_{b \in B} (A)_{-b}$
- **Dilation and erosion are duals of each other w.r.t. set complementation and reflection:**

$$A \ominus B = A^c \oplus \hat{B}$$

2 EXAMPLES OF EROSION

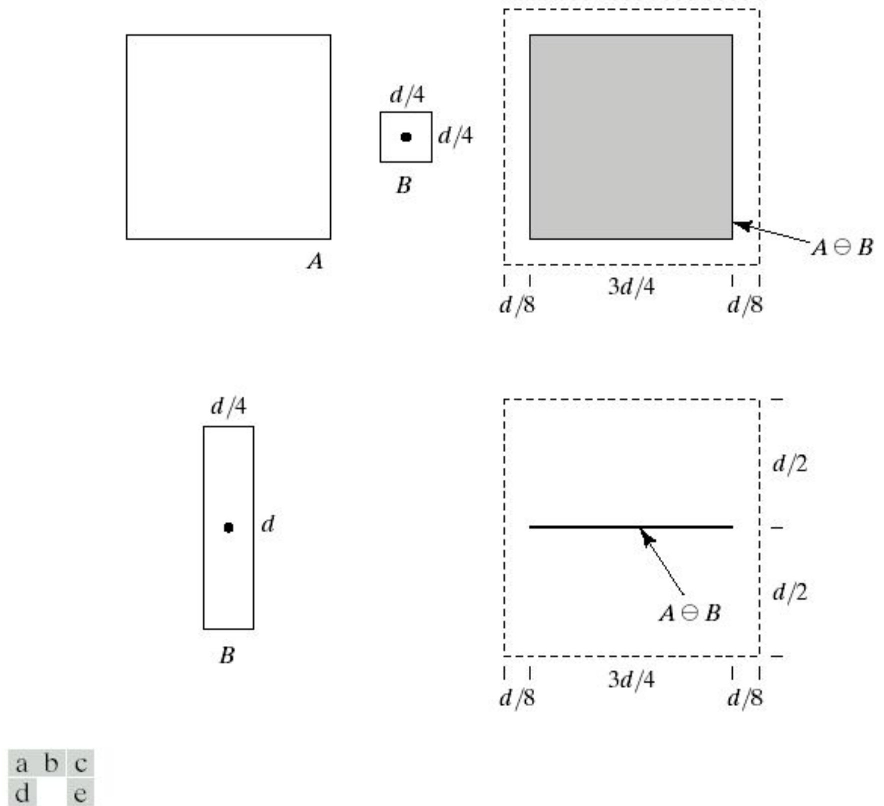
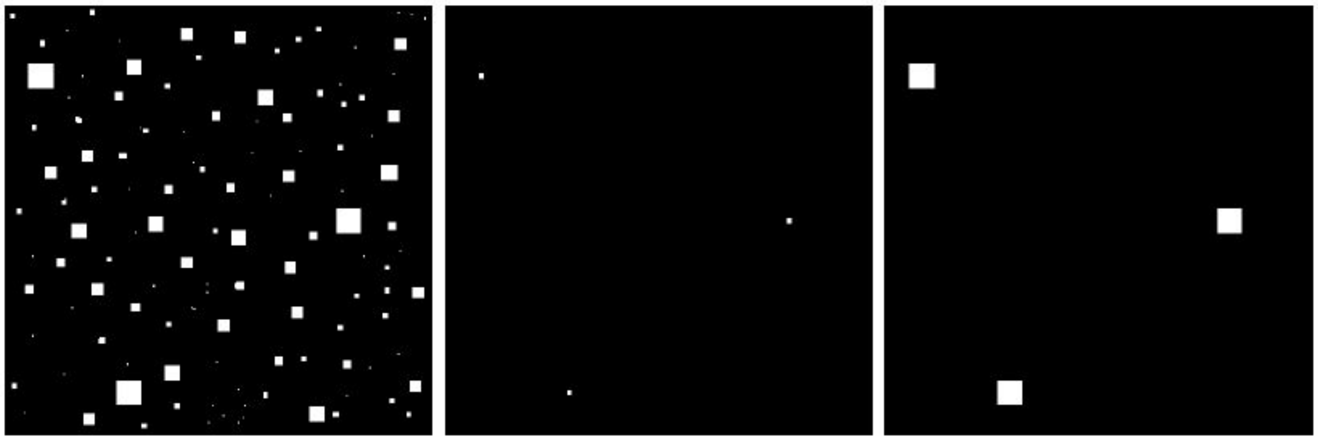


FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.

AN APPLICATION OF EROSION

In general, dilation does not <u>fully restore</u> eroded objects.



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Erosion

Structure element:
13X13 pixels
(all 1's)

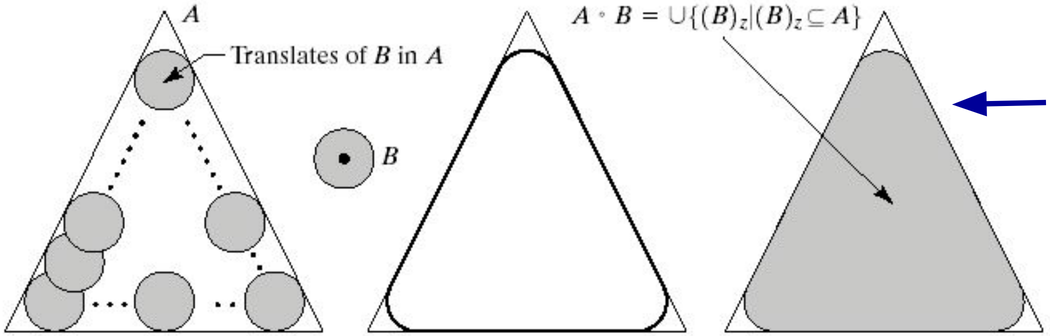
Dilation

Structure element:
13X13 pixels
(all 1's)

OPENING & CLOSING

- **opening** and **closing**: 2 other important morphological operations.
- **Opening *A* by B:** $A \circ B = (A \ominus B) \oplus B$
  - **Erosion of *A* by *B*, followed by a dilation of the result by *B*.**
  - **Generally smoothes the contour of an object**
  - **Breaks narrow isthmuses**
  - **Eliminates thin protrusions**
- **Closing of A by B:** $A \bullet B = (A \oplus B) \ominus B$
  - **Dilation of *A* by *B*, followed by an erosion of the result by *B*.**
  - **Also tends to smooth sections of contours.**
  - **Generally fuses narrow breaks and long thin gulfs**
  - **Eliminates small holes**
  - **Fills gaps in the contour**

GEOMETRIC INTERPRETATION OF OPENING



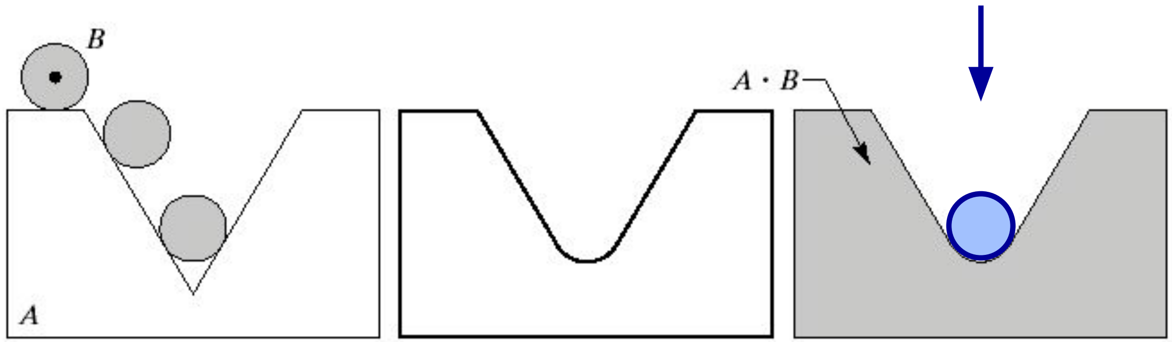
a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

the **union** **of all translates of *B* that fit into *A*.**

GEOMETRIC INTERPRETATION OF CLOSING

A point $w \in A \cdot B$ • $(B)_z \cap A \neq \varnothing$



a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

OPENING & CLOSING OPERATIONS

a
b c
d e
f g
h i

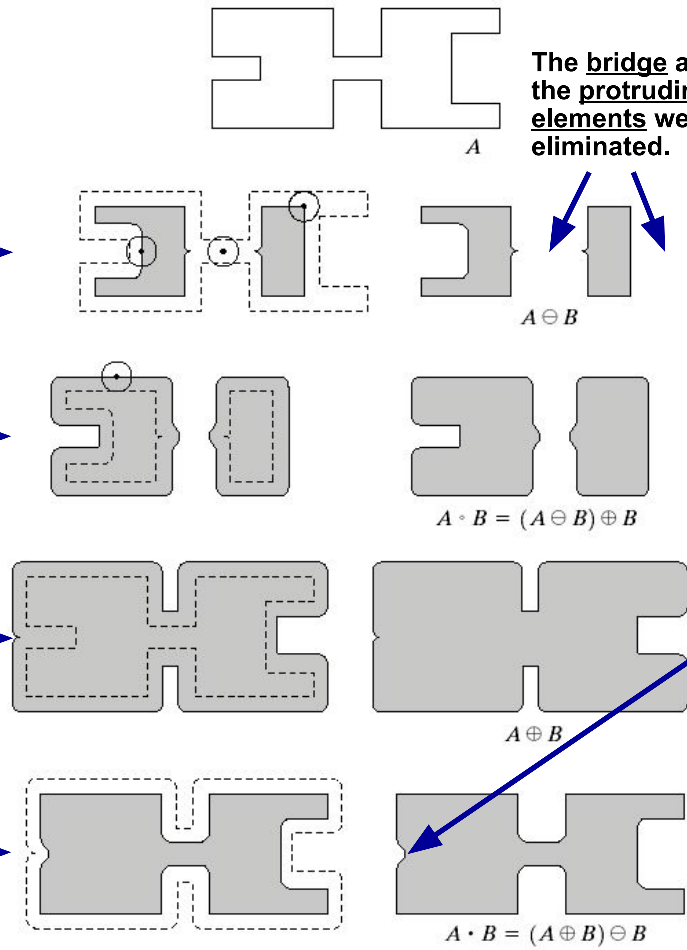
FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.

Erosion

Dilation

Dilation

Erosion



The <u>bridge</u> and the <u>protruding</u> <u>elements</u> were eliminated.

A circular structural element results in <u>smoothing</u> in parts of the object.

Outward pointing corners were <u>rounded</u>; inward pointing corners were <u>not</u> <u>affected</u>.

Opening of *A* by *B*

The size of the intrusion was significantly <u>reduced</u>.

Inward pointing corners were <u>rounded</u>; outward pointing corners were <u>not</u> <u>affected</u>.

Closing of *A* by *B*

PROPERTIES SATISFIED BY OPENING & CLOSING

- Opening and closing are **duals** of each other w.r.t. set complementation and reflection:

$$(A \bullet B)^c = (A^c \circ \hat{B})$$

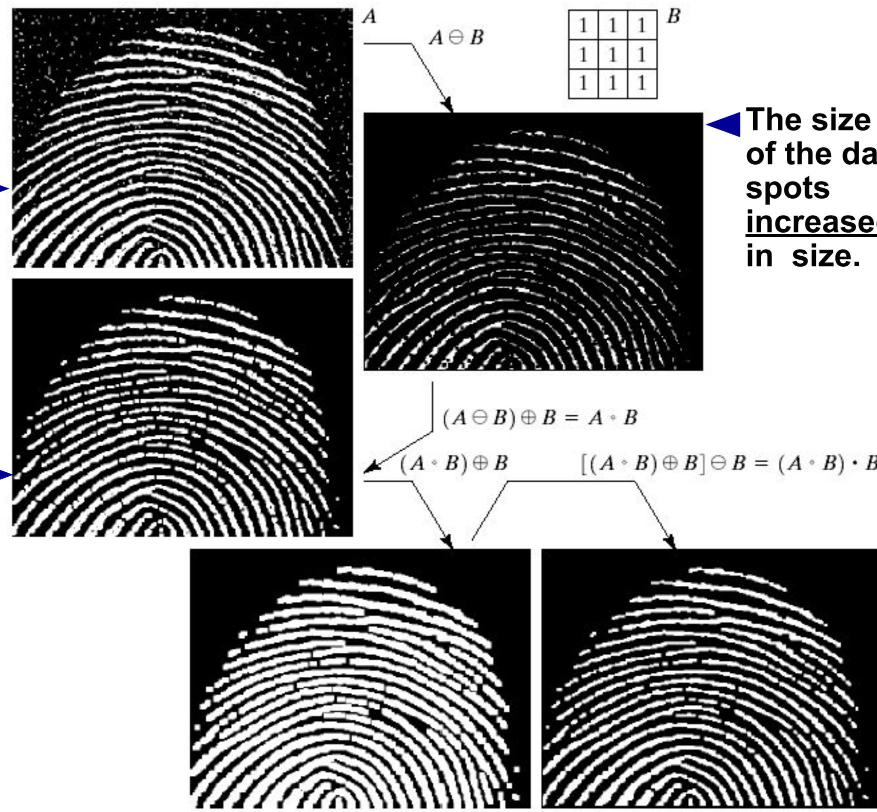
- The **opening** operation satisfies the following properties:
  - $A \circ B$ is a subset of A.
  - If $C \subseteq D$, then $(C \circ B) \subseteq (D \circ B)$.
  - $(A \circ B) \circ B = A \circ B$
- The **closing** operation satisfies the following properties:
  - A is a subset of $A \cdot B$.
  - If $C \subseteq D$, then $(C \cdot B) \subseteq (D \cdot B)$.
  - $(A \cdot B) \cdot B = A \cdot B$

A MORPHOLOGICAL FILTER: OPENING FOLLOWED BY CLOSING

Morphological operations can be used to construct filters.

The noise manifests itself as <u>light elements</u> on a <u>dark background</u> and as <u>dark elements</u> on the <u>light components</u> of the fingerprint.

The opening operation <u>reduced</u> the noise components in size or deleted them completely. However, it created <u>new gaps</u> between the fingerprint ridges.



The size of the dark spots <u>increased</u> in size.

a b
d c
e f

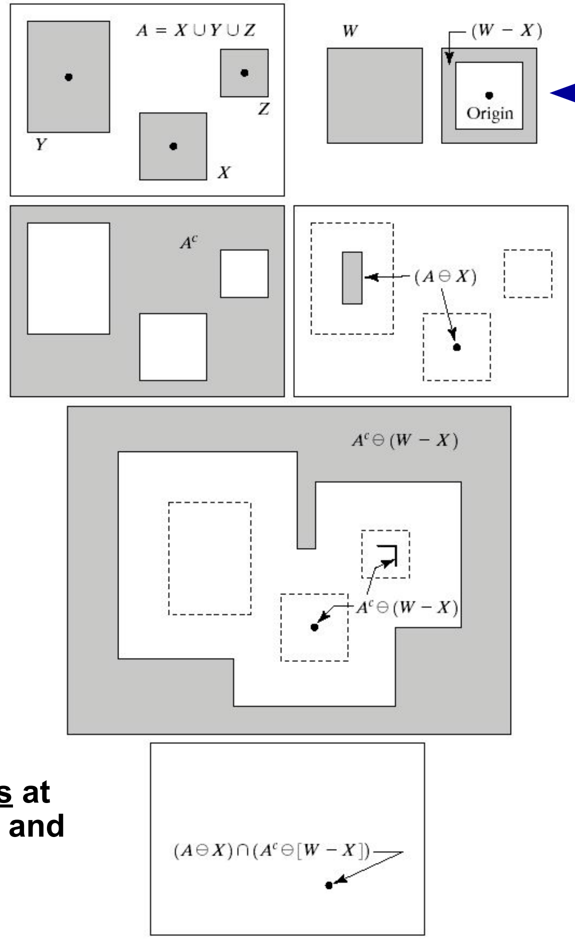
FIGURE 9.11
(a) Noisy image. (c) Eroded image. (d) Opening of A. (d) Dilation of the opening. (e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

The noise has been eliminated with <u>minimal</u> distortion to the fingerprint.

Most of the gaps were restored but the ridges were <u>thickened</u>.

HIT-OR-MISS TRANSFORMATION

Set A is the union of 3 **disjoint** sets



Local background of X w.r.t. W

$A \ominus X$: set of all locations of the origin of X at which X found a match (hit) in A.

$A^c \ominus (W-X)$: set of all locations of the origin of $(W-X)$ at which $(W-X)$ found a match (hit) in A^c.

Generalized notation:

$B = (B_1, B_2)$

$B_1 = X$

$B_2 = (W - X)$

$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$

Contains all the origin points at which B_1 found a match in A and B_2 found a match in A^c.

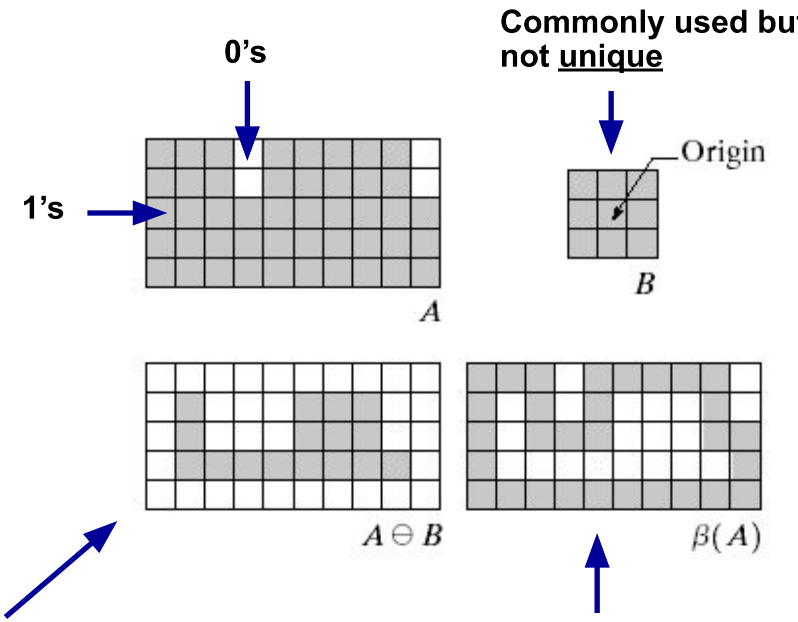
SOME BASIC MORPHOLOGICAL ALGORITHMS

- Boundary extraction
- Region filling
- Extraction of connected components
- Convex hull
- Thinning
- Thickening
- Skeletons
- Pruning

BOUNDARY EXTRACTION

a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.



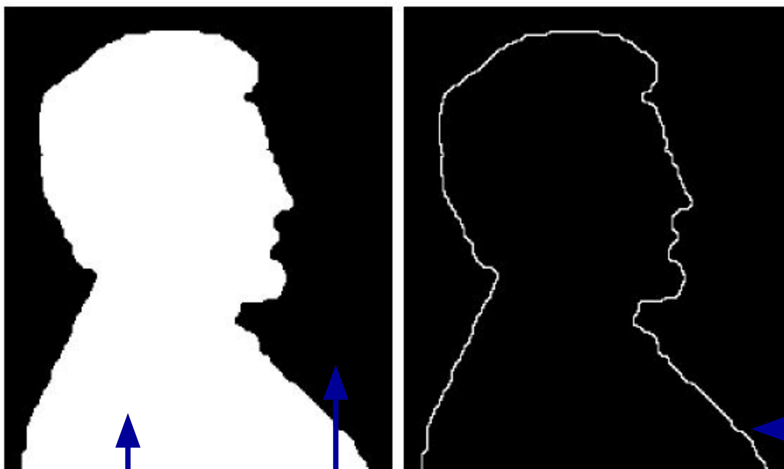
$$\beta(A) = A - (A \ominus B)$$

When the origin of B is on the edge of the set A, part of B may be <u>outside</u> the image.

How do we handle this <u>situation</u>?

Normal treatment: Assume that values outside the borders of the image are 0.

AN APPLICATION OF BOUNDARY EXTRACTION



a b

FIGURE 9.14 (a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

The boundary is 1 pixel thick

1's **0's**

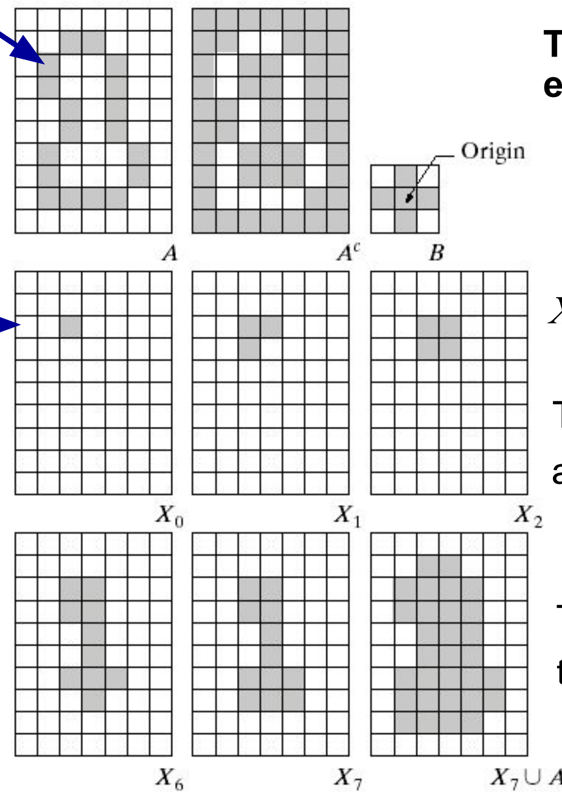
1	1	1
1	1	1
1	1	1

Structural element

REGION FILLING

A subset whose elements are **8-connected** boundary points of a region.

$X_0 = p$



a b c
d e f
g h i

FIGURE 9.15 Region filling. (a) Set A. (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

The objective is to fill the entire region with 1's.

$$X_k = (X_{k-1} \oplus B) \cap A^c, k = 1,2,3,...$$

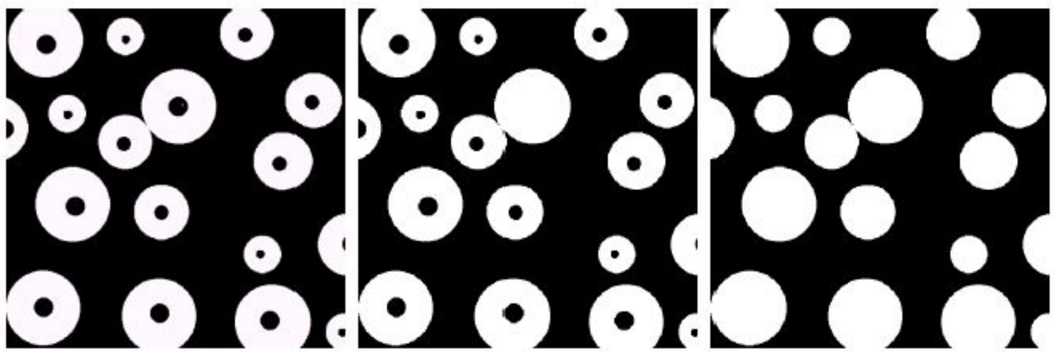
The algorithm terminates at step k when $X_k = X_{k-1}$

The union of X_k and A contains the filled region and its boundary.

AN APPLICATION OF REGION FILLING

Such an image may be the result of <u>thresholding into 2 levels</u> a scene containing polished spheres.

We will eliminate the <u>reflections</u> by region filling.



a b c

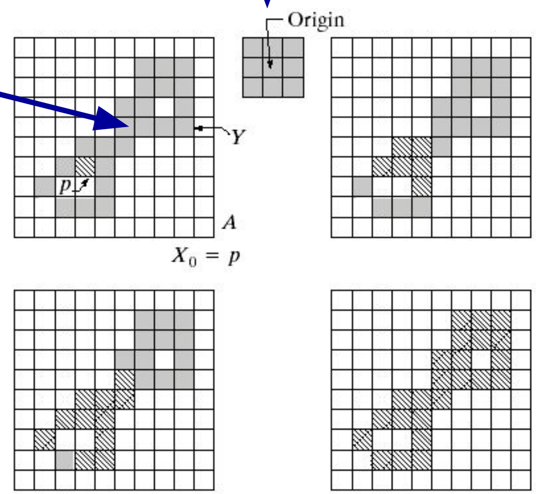
FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

EXTRACTION OF CONNECTED COMPONENTS

Extraction of connected components in a binary image is central to many automated image analysis applications.

The shape of B assumes 8-connectivity between pixels

***Y* is a connected component in *A*.**



a b c
d e

FIGURE 9.17 (a) Set A showing initial point p (all shaded points are valued 1, but are shown different from p to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

$$X_k = (X_{k-1} \oplus B) \cap A, \; k = 1,2,3,\ldots$$

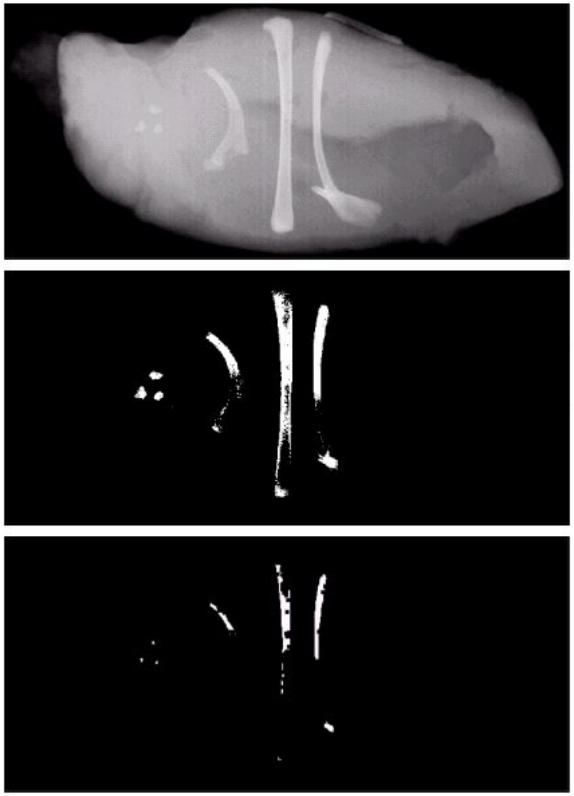
$$X_0 = p$$

The algorithm terminates at step k when $X_k = X_{k-1}$.

$$Y = X_k$$

AN APPLICATION OF CONNECTED COMPONENT EXTRACTION

Connected components are frequently used for <u>automated inspection</u>.



a
b
c d

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1's. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Analysis of the size of the remaining objects.

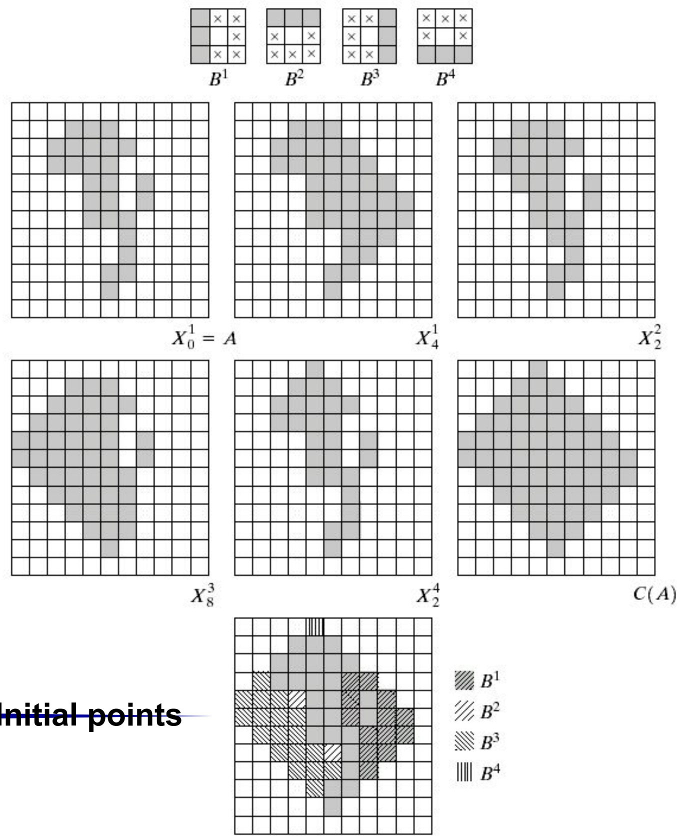
Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

4 of the connected components are dominant in size.

CONVEX HULL

- A set A is said to be **convex** if the straight line segment joining any 2 points lies entirely within A.
- The **convex hull** H of an arbitrary set S is the smallest convex set containing S.
- The set difference H-S is called the **convex deficiency** of S.
- The convex hull and convex deficiency are useful for **object description**.
- The procedure for obtaining the convex hull, $C(A)$, of a set S:
  - Starting with X_0^i, iteratively apply the hit-or-miss transform to A with B^i. When the iteration converges, perform the union with A, and call the result D^i.
  - $C(A)$ = union of all D^i's.

CONVEX HULL: AN EXAMPLE



a
b c d
e f g
h

FIGURE 9.19 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.

Algorithm for obtaining the convex hull, *C*(*A*), of a set *A*:

$B^i, i = 1,2,3,4$ **: 4 structuring elements**

$$X_k^i = (X_{k-1} \circledast B^i) \cap A,$$

$i = 1,2,3,4$, and $k = 1,2,3,\ldots$

$$X_0 = A$$

$D^i = X_{conv}^i$ **with** $X_k = X_{k-1}$

$$C(A) = \bigcup_{i=1}^{4} D^i$$

$$X_0^1 = A$$
$$X_0^2 = A$$
$$X_0^3 = A$$
$$X_0^4 = A$$

Initial points

SHORTCOMING OF THE ALGORITHM FOR OBTAINING THE CONVEX HULL

Shortcoming of the algorithm: the convex hull can grow beyond the min dimensions required to guarantee convexity.

A simple approach to reduce this effect: limit growth so that it does not extend past the vertical and horizontal dimensions of the original set of points.

Boundaries of greater complexity can be used to limit growth even further in images with more detail, e.g., the max dimensions of the original set of points along the horizontal, vertical, and diagonal directions can be used.

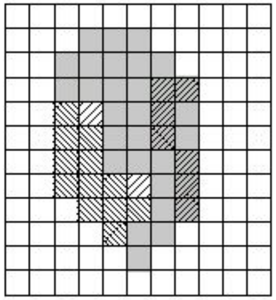


FIGURE 9.20 Result of limiting growth of convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

EXTENSIONS TO GRAY SCALE IMAGES

- The following basic operations will be extended to gray scale images:
  - Dilation
  - Erosion
  - Opening
  - Closing
- These operations will be used to develop several morphological algorithms.
  - Boundary extraction
  - Region partitioning
  - Smoothing
  - Sharpening
- $f(x,y)$: input image, $b(x,y)$: a structuring element

GRAY SCALE DILATION

$$(f \oplus b)(s,t) = \max\{f(s-x,t-y)+b(x,y) \mid (s-x),(t-y) \in D_f; (x,y) \in D_b\}$$

D_f : domain of f

D_b : domain of b

f **and** ***b*** **are <u>functions</u>, and not <u>sets</u>.**

Gray scale: $(s-x),(t-y) \in D_f; (x,y) \in D_b$

Binary: the 2 sets have to overlap by at least 1 element

analogous

The <u>general effect</u> of performing dilation on a gray scale image:

- **The values of *b* > 0 • the output image tends to be <u>brighter</u>.**
- **<u>Dark details</u> are either <u>reduced</u> or <u>eliminated</u>, depending on how their values and shapes relate to *b* used for dilation.**

1-D EXAMPLE OF GRAY SCALE DILATION

$$(f \oplus b)(s) = \max\{f(s-x)+b(x) \mid (s-x) \in D_f ; x \in D_b\}$$

Unlike the binary case, ***f***, not ***b***, is shifted.

Conceptually, ***f* sliding by *b*** is **not different** from ***b* sliding by *f***.

The actual **mechanics** of gray scale dilation is easier to visualize **if *b* is the function that slides past *f***.

At each position of ***b***, the value of dilation at that point is the **max of *f*+*b*** in the interval spanned by ***b***.

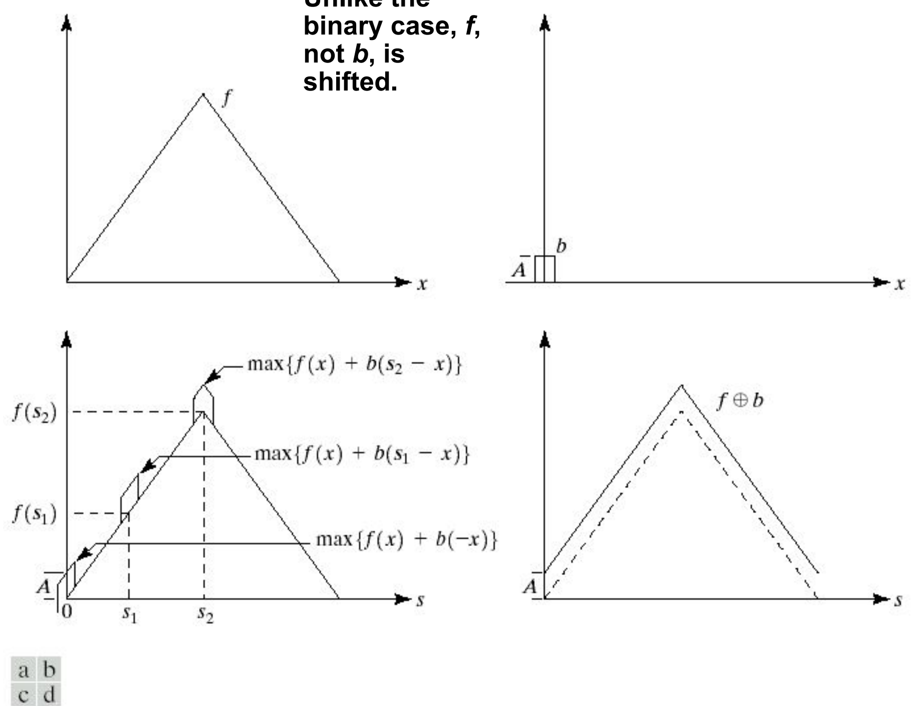


FIGURE 9.27 (a) A simple function. (b) Structuring element of height A. (c) Result of dilation for various positions of sliding b past f. (d) Complete result of dilation (shown solid).

GRAY SCALE EROSION

$$(f \ominus b)(s,t) = \min\{f(s+x,t+y)+b(x,y) \mid (s+x),(t+y) \in D_f; (x,y) \in D_b\}$$

D_f : domain of f

D_b : domain of b

Gray scale: $(s+x),(t+y) \in D_f; (x,y) \in D_b$

Binary: b has to be completely contained by A.

~~**analogous**~~

The general effect of performing erosion on a gray scale image:

- **The values of *b* > 0 • the output image tends to be darker.**
- **Bright details that are smaller in area than the structuring element are reduced, with the degree of reduction determined by the gray level values surrounding the bright detail, and by the shape and values of *b*.**

1-D EXAMPLE OF GRAY SCALE EROSION

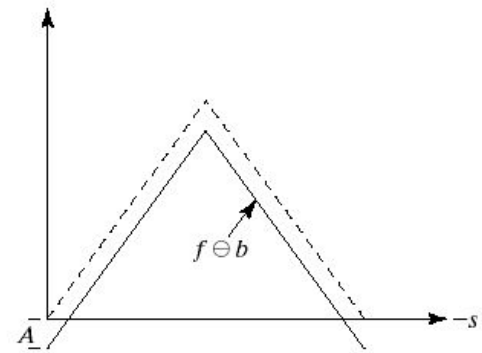
$$(f \ \Theta \ b)(s) = \max\{f(s+x) - b(x) \mid (s+x) \in D_f ; x \in D_b\}$$

Conceptually, <u>*f* sliding by *b*</u> is not different from <u>*b* sliding by *f*</u>.

Unlike the binary case, *f*, not *b*, is shifted.

At each position of *b*, the value of erosion at that point is the min of *f-b* in the interval spanned by *b*.

FIGURE 9.28 Erosion of the function shown in Fig. 9.27(a) by the structuring element shown in Fig. 9.27(b).

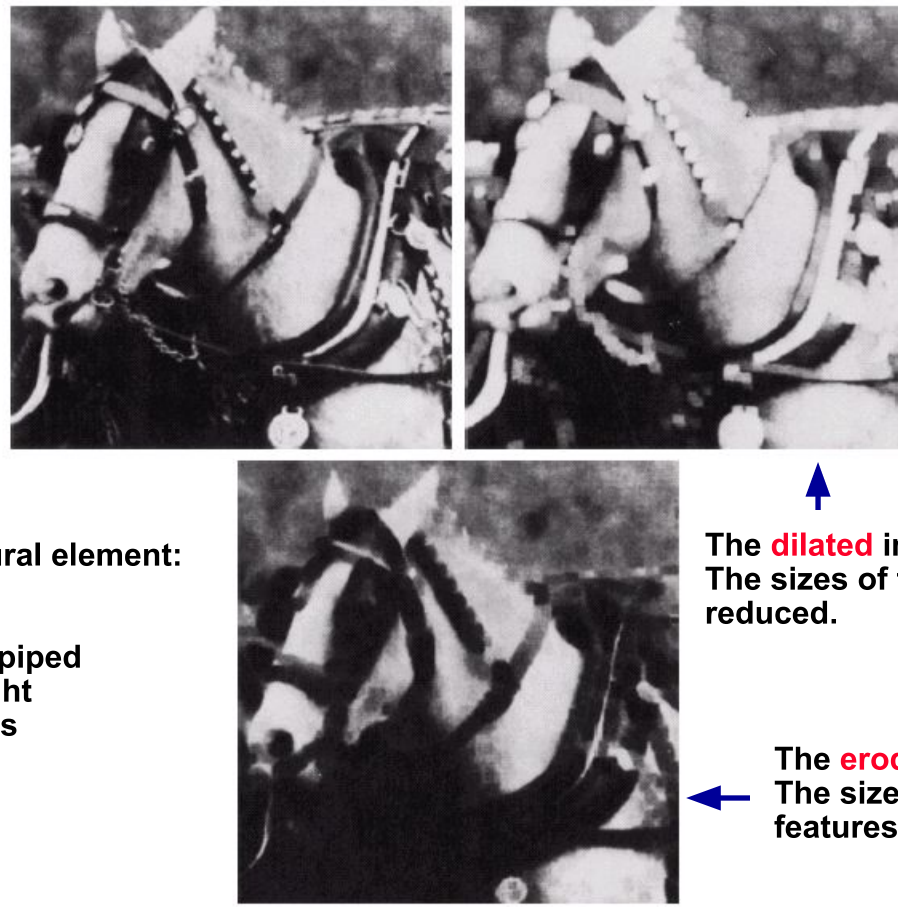


$$(f \ \mathrm{O} \ b)^c(s,t) = (f^c \oplus \hat{b})(s,t)$$

$$f^c = -f(x,y) \ \& \ \hat{b} = b(-x,-y)$$

<u>Gray scale dilation</u> and erosion are duals w.r.t. <u>function complementation</u> and <u>reflection</u>.

AN EXAMPLE OF GRAY SCALE DILATION & EROSION



a b
c

FIGURE 9.29 (a) Original image. (b) Result of dilation. (c) Result of erosion. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

The structural element:

- **Flat-top**
- **Parallelepiped**
- **Unit height**
- **5x5 pixels**

**The dilated image is <u>brighter</u>.
The sizes of the dark features are reduced.**

**The eroded image is darker.
The sizes of the small, bright features are reduced.**

OPENING AND CLOSING FOR GRAY SCALE IMAGES

$f \circ b = (f \ominus b) \oplus b$: **The opening of image *f* by subimage *b***

$f \bullet b = (f \oplus b) \ominus b$: **The closing of image *f* by subimage *b***

$(f \bullet b)^c = f^c \circ \hat{b}$: **The opening & closing of gray scale images are duals w.r.t. <u>function complementation</u> and <u>reflection</u>.**

Opening and closing of images have a simple geometric interpretation:

Suppose we view an image function *f*(*x*,*y*) in 3D perspective.

The image appears as a <u>discrete</u> surface: *f* at (*x*,*y*).

<u>Opening</u>: rolling a ball against the underside of the surface.

<u>Closing</u>: rolling a ball on top of the surface.

GEOMETRIC INTERPRETATION OF OPENING & CLOSING

To simplify the illustration, a scan line of a gray scale image is shown as a <u>continuous</u> function.

Opening: rolling a ball against the underside of the surface.

All the peaks that were narrow w.r.t. the diameter of the ball were reduced in amplitude and sharpness.

Closing: rolling a ball on top of the surface.

The peaks are left in their original form.

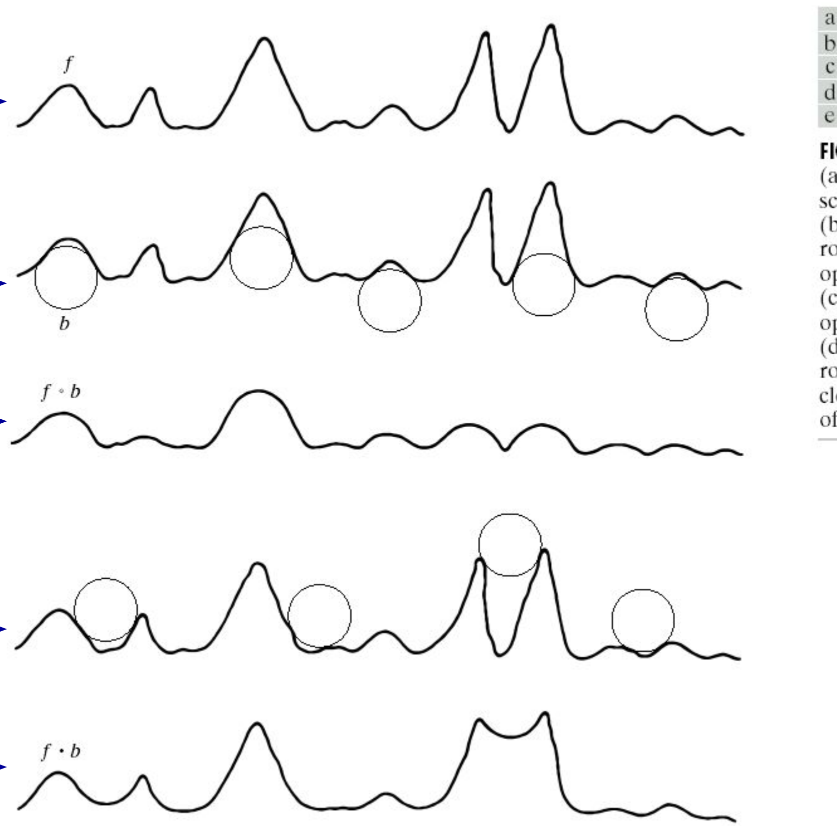


FIGURE 9.30 (a) A gray-scale scan line. (b) Positions of rolling ball for opening. (c) Result of opening. (d) Positions of rolling ball for closing. (e) Result of closing.

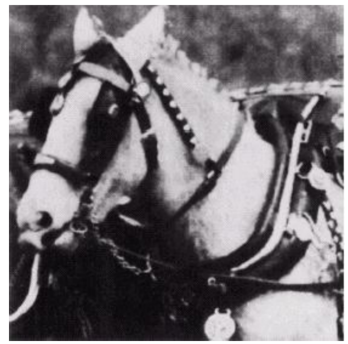
OPENING AND CLOSING OF A GRAY SCALE IMAGE

$$f \circ b = (f \ominus b) \oplus b$$

The initial <u>erosion</u> removes the small details but it also darkens the image.

The subsequent <u>dilation</u> increases the overall intensity without reintroducing the details removed by erosion.

Original image

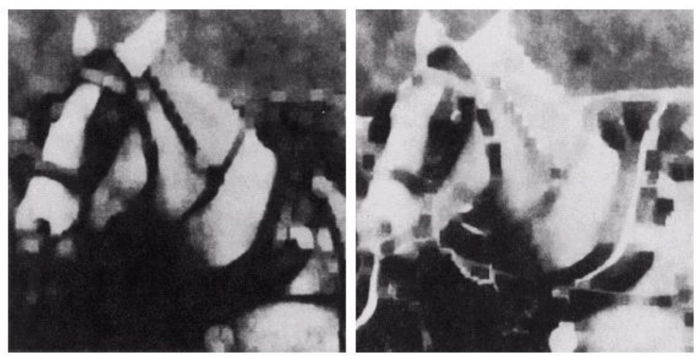


$$f \bullet b = (f \oplus b) \ominus b$$

The initial <u>dilation</u> removes the dark details but it also brightens the image.

The subsequent <u>erosion</u> darkens the image without reintroducing the details removed by dilation.

Opening is generally used to remove <u>small light details</u> from an image while leaving the <u>overall gray levels and larger bright features</u> relatively undisturbed.



Closing is generally used to remove <u>dark details</u> from an image while leaving <u>bright features</u> relatively undisturbed.

a b

FIGURE 9.31 (a) Opening and (b) closing of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

MORPHOLOGICAL SMOOTHING

One way to achieve smoothing is to perform a morphological opening followed by a closing.

The net result of these 2 operations is to remove or attenuate both bright and dark artifacts or noise.

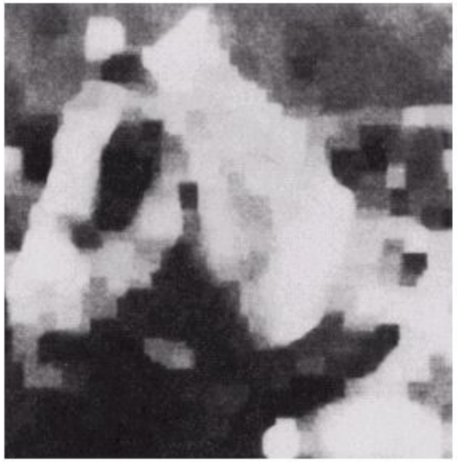


FIGURE 9.32 Morphological smoothing of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

The structural element:

- Flat-top
- Parallelepiped
- Unit height
- 5x5 pixels

MORPHOLOGICAL GRADIENT

dilation erosion

$$g = (f \oplus b) - (f \ominus b)$$

The **morphological gradient** highlights sharp gray level transitions in the input image.

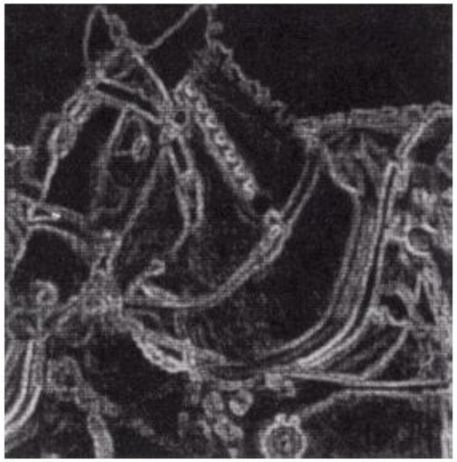


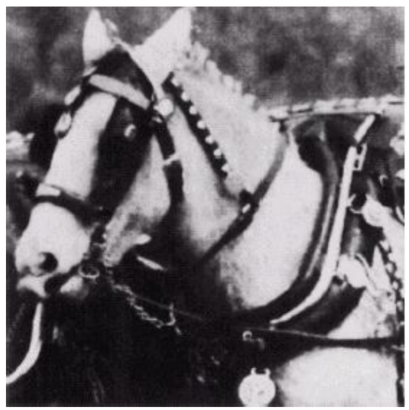
FIGURE 9.33 Morphological gradient of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

The structural element:

- Flat-top
- Parallelepiped
- Unit height
- 5x5 pixels

TOP-HAT TRANSFORMATION

Original image



$$h = f - (f \circ b)$$

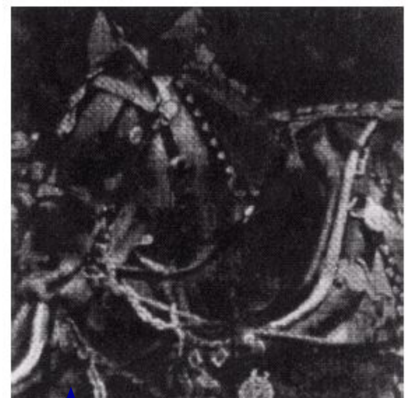


FIGURE 9.34 Result of performing a top-hat transformation on the image of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Note the enhancement of detail in the background region below the lower part of the horse's head.

TEXTURAL SEGMENTATION

Closing tends to remove dark details from an image.

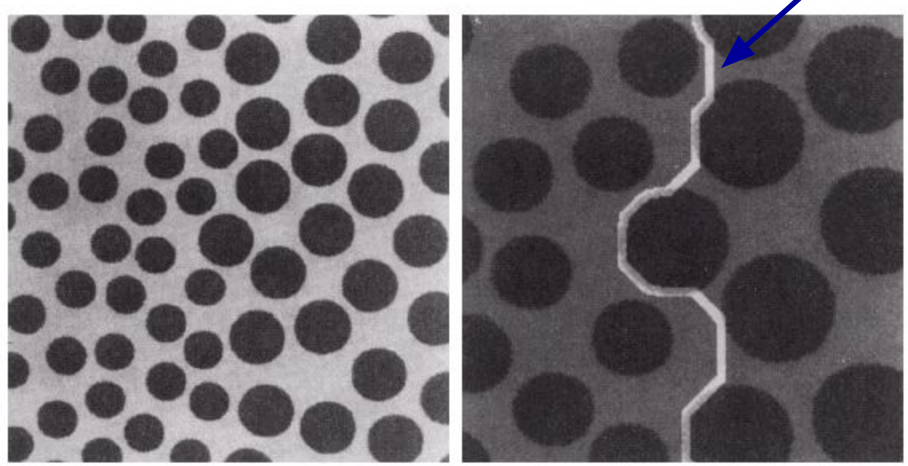
Procedure:

1. **Close the image by using successively larger structuring elements.**
2. **When the size of the structuring element corresponds to that of the small blobs, the small blobs are removed from the image, leaving a light background in the area previously occupied by them.**
3. **At this point, only the larger blobs and the light background on the left and between the large blobs themselves remain.**
4. **A single opening is performed with a structuring element that is large in relation to the separation between the large blobs.**
5. **This operation removes the light patches between the large blobs, leaving a large region on the right consisting of the large dark blobs and the now equally dark patches between these blobs.**
6. **The process has produced a light region on the left and a dark region on the right.**
7. **A simple threshold yields the boundary between the 2 textural regions.**

Boundary superimposed on the original image

a b

FIGURE 9.35 (a) Original image. (b) Image showing boundary between regions of different texture. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)



GRANULOMETRY

Granulometry: a field that deals principally with determining the size distribution of particles in an image.

Procedure:

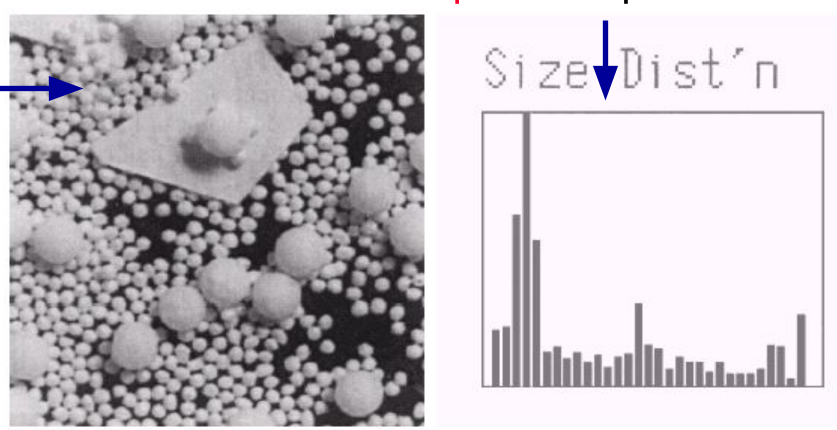
1. **Opening operations with structural elements of increasing size are performed on the original image.**
2. **The difference between the original image and its opening is computed after each pass when a different structural element is completed.**
3. **At the end of the process, these differences are normalized, and then used to construct a histogram of particle size distribution.**

The approach is based on the idea that opening operations of a particular size have the most effect on regions of the input image that contain particles of similar size. Thus, a measure of the relative number of such particles is obtained by computing the difference between the input and output images.

Indicates the presence of 3 predominant particle sizes.

Light objects of 3 different sizes.

The objects are <u>overlapping</u> and <u>too cluttered</u> to enable detection of individual particles.



a b

FIGURE 9.36 (a) Original image consisting of overlapping particles; (b) size distribution. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)