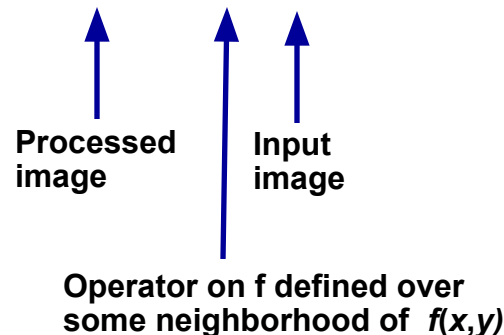


CHAPTER 3

IMAGE ENHANCEMENT IN THE SPATIAL DOMAIN

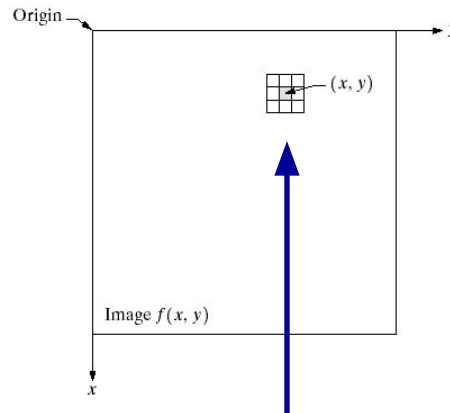
CHAPTER 3: IMAGE ENHANCEMENT IN THE SPATIAL DOMAIN

- ❑ Principal objective: to process an image so that the result is more suitable than the original image for a *specific* application.
- ❑ There is no general theory of image enhancement.
- ❑ The viewer is the ultimate judge of how well a particular method works.
- ❑ Visual evaluation of image quality is a highly subjective process!
- ❑ *Spatial domain*: aggregate of pixels composing an image.
- ❑ Spatial domain processes: $g(x,y) = T[f(x,y)]$



DEFINING A NEIGHBORHOOD

FIGURE 3.1 A 3×3 neighborhood about a point (x, y) in an image.

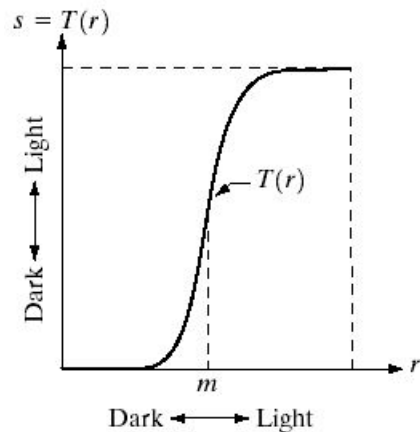


A square or rectangle is commonly used in defining the neighborhood about a point (x, y) .

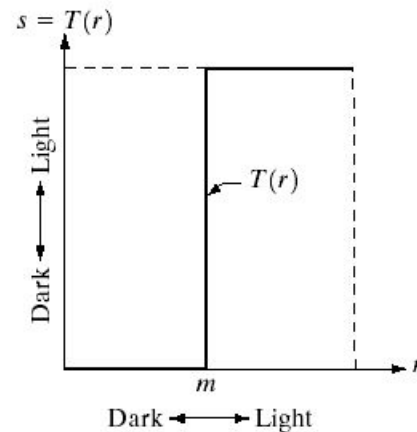
Operator T is applied at each location (x, y) to produce the output g at that location.

Other neighborhood shapes (e.g., approximation to a circle) are sometimes used.

GRAY LEVEL TRANSFORMATION FUNCTION



Contrast stretching:
Produces an image of higher contrast than the original.



Thresholding function:
Produces a binary image.

a b

FIGURE 3.2 Gray-level transformation functions for contrast enhancement.

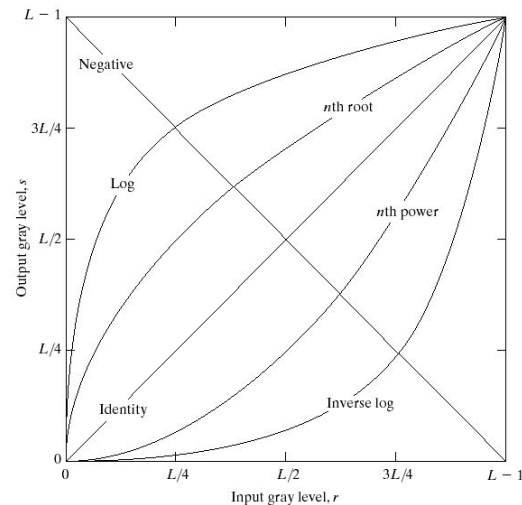
Neighborhood of size 1x1 => T is a gray level transformation function.

Larger neighborhoods => masks (filters)

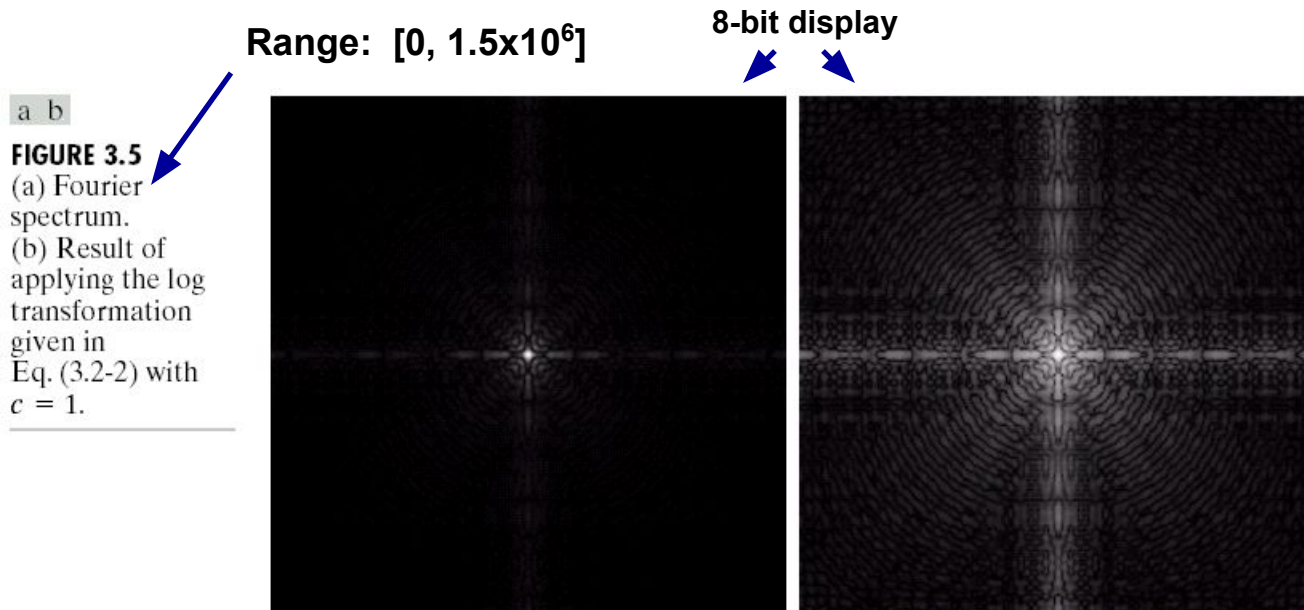
BASIC GRAY LEVEL TRANSFORMATIONS

- ❑ Among the simplest image enhancement techniques.
- ❑ $s = T(r)$: T maps a pixel value r into a pixel value s .
- ❑ Three basic types of transformations
 - ❑ **Linear** (negative and identity)
 - ❑ **Logarithmic** (log and inverse-log)
 - ❑ **Power law** (n th power and n th root)

FIGURE 3.3 Some basic gray-level transformation functions used for image enhancement.



LOG TRANSFORMATIONS



$$s = c \log (1 + r), r \geq 0$$

Maps a narrow range of low gray-level values into a wider range.
Maps a wide range of high gray-level values into a narrower range.

Inverse log transformation?

Compresses the dynamic range of image with large variations in pixel values.

POWER LAW TRANSFORMATIONS

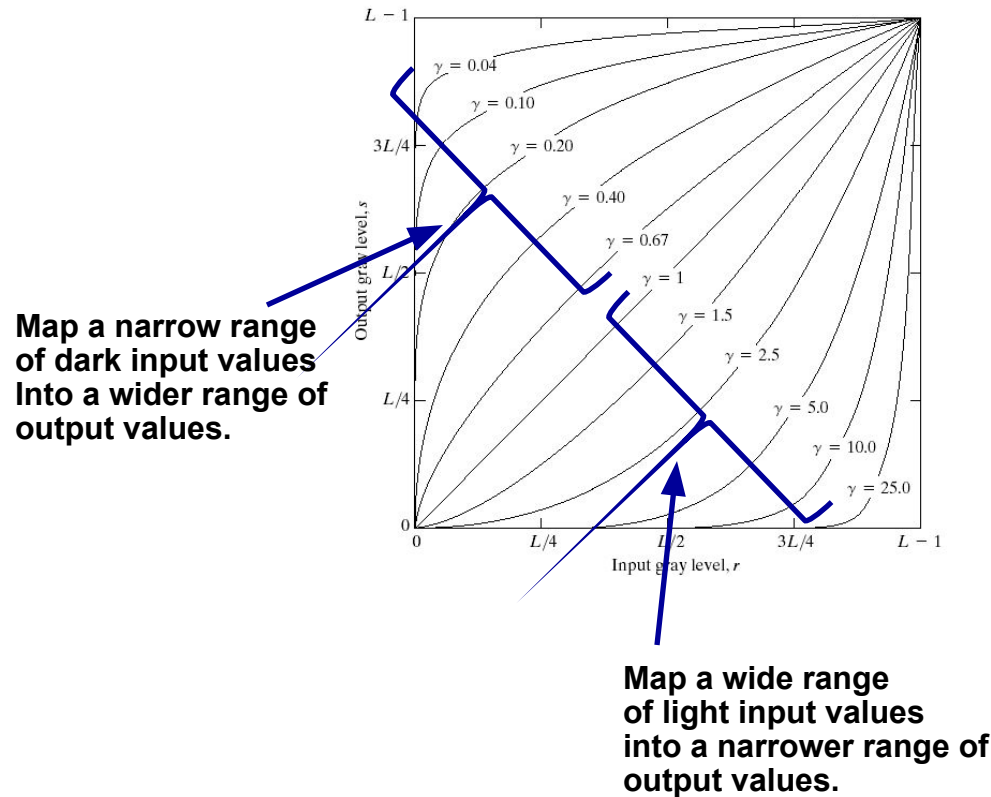


FIGURE 3.6 Plots of the equation $s = cr^\gamma$ for various values of γ ($c = 1$ in all cases).

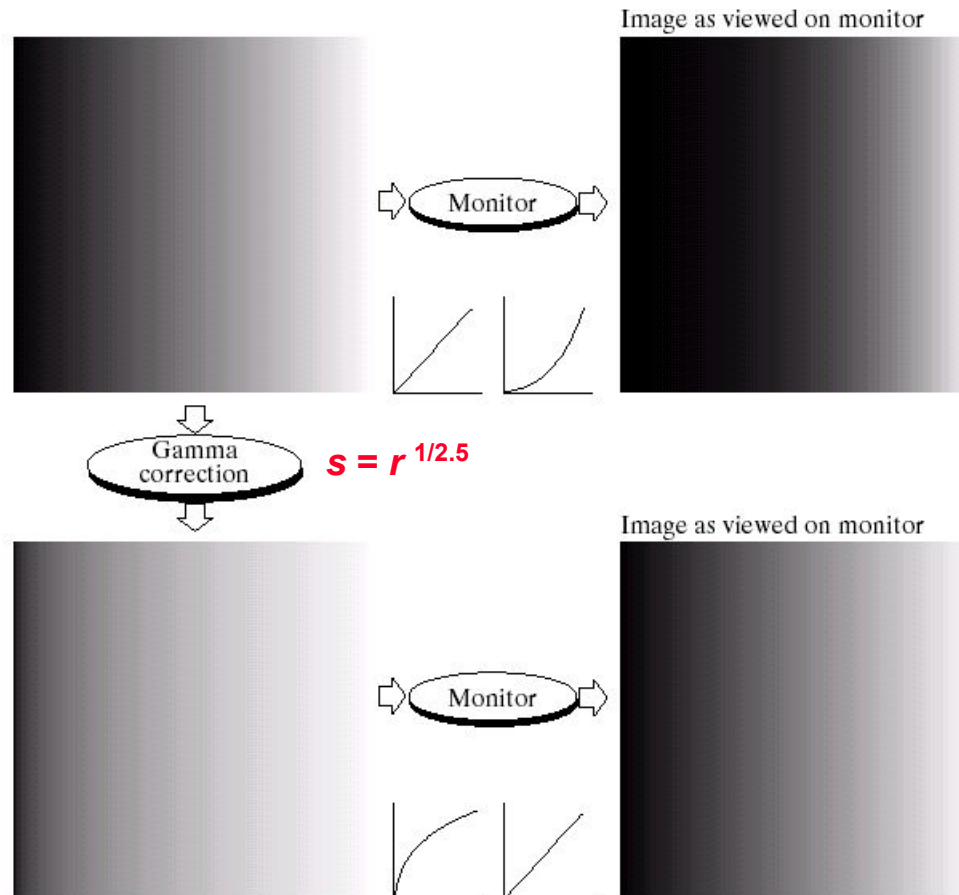
$$s = cr^\gamma \text{ or } c(r + \epsilon)^\gamma, c \text{ and } \gamma > 0$$

GAMMA CORRECTION

a b
c d

FIGURE 3.7

(a) Linear-wedge gray-scale image.
(b) Response of monitor to linear wedge.
(c) Gamma-corrected wedge.
(d) Output of monitor.

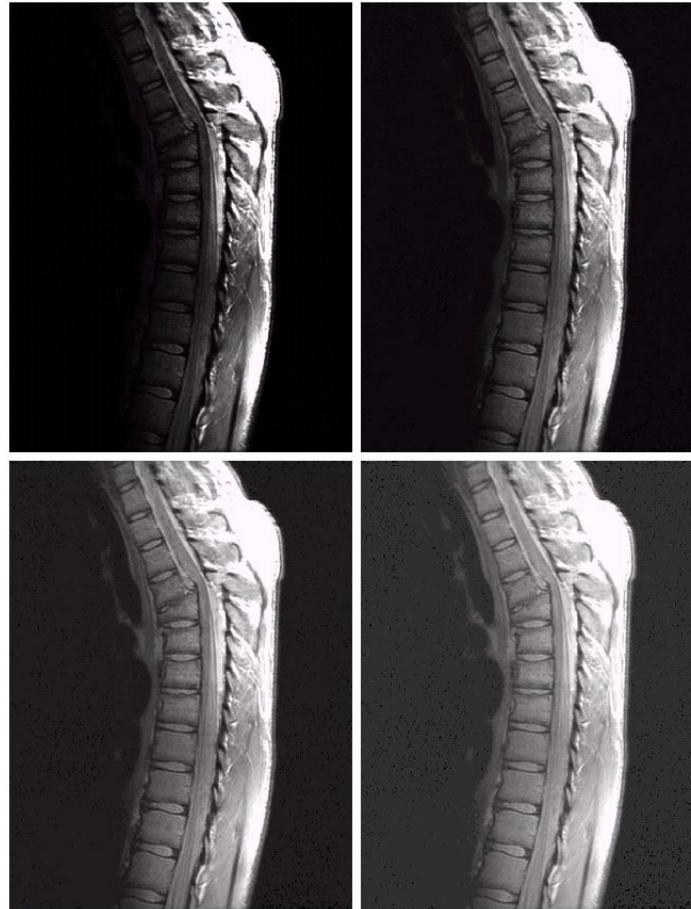


A variety of devices for image capture, printing and display respond according to power law.

Such systems tend to produce images that are darker than intended.

CONTRAST MANIPULATION

Original image
is predominantly
dark!



a b
c d

FIGURE 3.8

(a) Magnetic resonance (MR) image of a fractured human spine.

(b)-(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6, 0.4$, and 0.3 , respectively. (Original image for this example courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

$$s = r^{0.6}$$

$$s = r^{0.4}$$

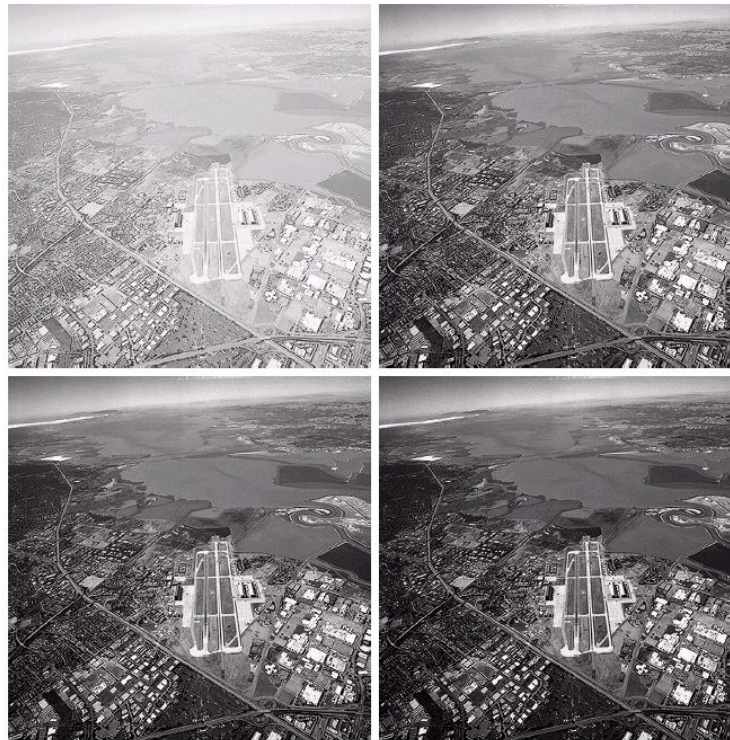
$$s = r^{0.3}$$

CONTRAST MANIPULATION

Original image
has a washed-out
appearance!

a b
c d

FIGURE 3.9
(a) Aerial image.
(b)–(d) Results of
applying the
transformation in
Eq. (3.2-3) with
 $c = 1$ and
 $\gamma = 3.0, 4.0,$ and
 5.0 , respectively.
(Original image
for this example
courtesy of
NASA.)



$$s = r^{3.0}$$

$$s = r^{4.0}$$

$$s = r^{5.0}$$

PIECEWISE-LINEAR TRANSFORMATIONS

- ❑ A complementary approach to the previous methods.
- ❑ Advantage: The form of piecewise functions can be arbitrarily complex.
- ❑ Disadvantage: requires considerable more user input.
- ❑ **Contrast stretching**
 - ❑ Increases the dynamic range of gray levels in the image.
- ❑ **Gray-level slicing**
 - ❑ Highlights a specific range of gray levels in the image.
- ❑ **Bit-plane slicing**
 - ❑ Highlights the contribution made by specific bits.
 - ❑ If each pixel is represented by 8 bits, the image is sliced into 8 planes.

CONTRAST STRETCHING

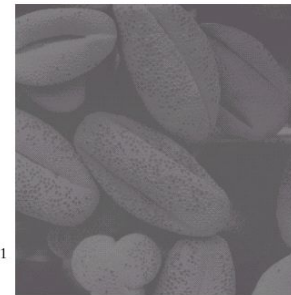
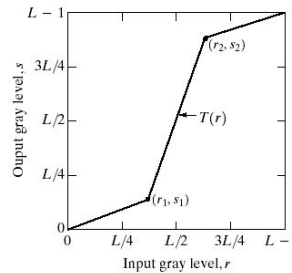
Typical transformation

•
 (r_1, s_1) and (r_2, s_2)
 control the shape of
 the function.

$$\left. \begin{array}{l} r_1 = s_1 \\ r_2 = s_2 \end{array} \right\} \text{linear}$$

$$\left. \begin{array}{l} r_1 = r_2 \\ s_1 = 0 \\ s_2 = L - 1 \end{array} \right\} \text{threshold}$$

$r_1 \leq r_2$ and $s_1 \leq s_2$:
 single valued
 monotonically
 increasing function



a b
c d

FIGURE 3.10
 Contrast stretching.
 (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)



$$\begin{array}{l} (r_1, s_1) = (r_{\min}, 0) \\ (r_2, s_2) = (r_{\max}, L - 1) \end{array}$$

$$\begin{array}{l} (r_1, s_1) = (m, 0) \\ (r_2, s_2) = (m, L - 1) \end{array}$$

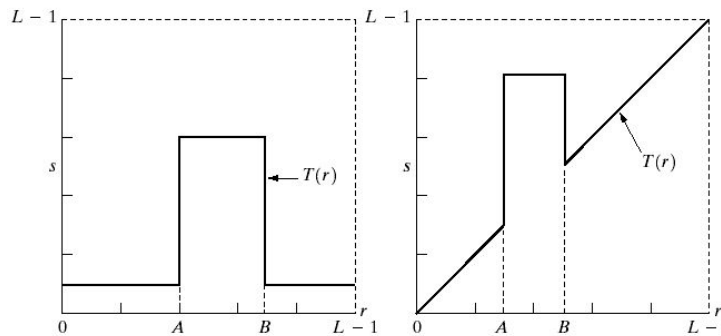
mean gray level

GRAY-LEVEL SLICING

Two approaches:

Display a high level for all gray levels in the range of interest and a low value for all other gray levels.

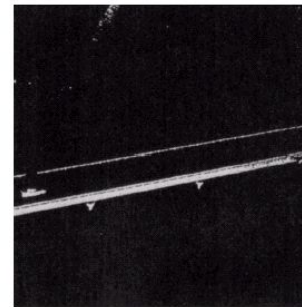
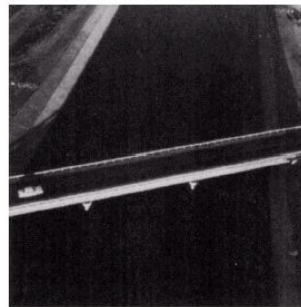
Brighten the desired range of gray levels but preserve the background and gray level tonalities.



a b
c d

FIGURE 3.11

(a) This transformation highlights range $[A, B]$ of gray levels and reduces all others to a constant level.
(b) This transformation highlights range $[A, B]$ but preserves all other levels.
(c) An image.
(d) Result of using the transformation in (a).



A gray scale image

Transformation (a) applied.

BIT-PLANE SLICING

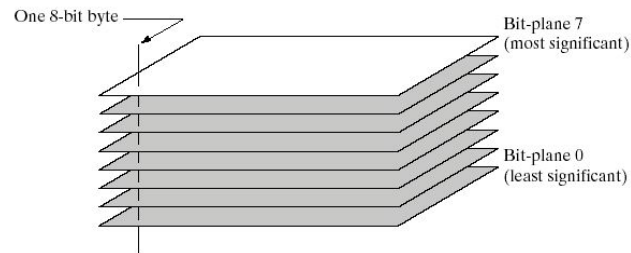


FIGURE 3.12
Bit-plane
representation of
an 8-bit image.

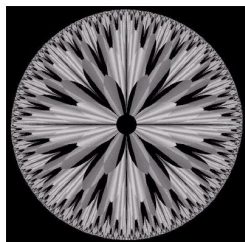
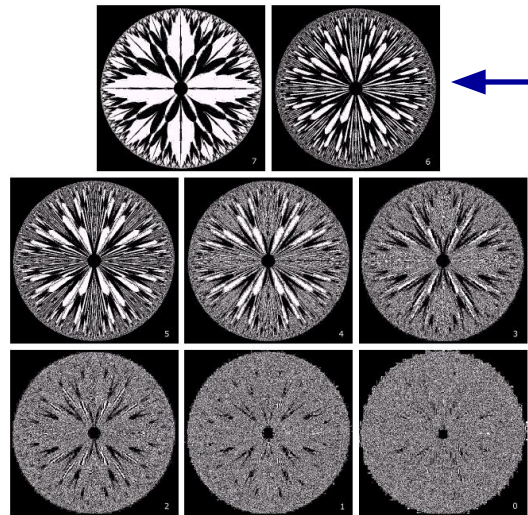
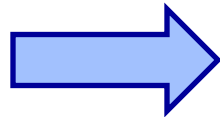


FIGURE 3.13 An 8-bit fractal image. (A fractal is an image generated from mathematical expressions). (Courtesy of Ms. Melissa D. Binde, Swarthmore College, Swarthmore, PA.)

**Slicing into
8 bit planes**



**Higher-order bits contain
the majority of the visually
significant data.**

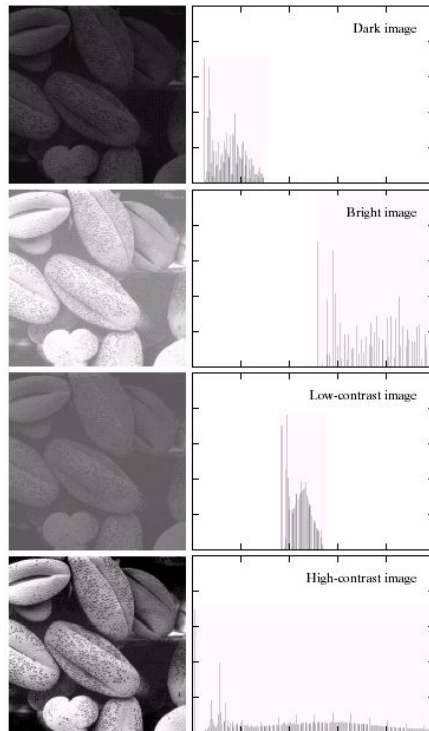
**Other bit planes
contain subtle
details.**

FIGURE 3.14 The eight bit planes of the image in Fig. 3.13. The number at the bottom, right of each image identifies the bit plane.

HISTOGRAM PROCESSING

- ❑ Histogram of a digital image
 - ▢ $h(r_k) = n_k$
 - ▢ r_k : k^{th} gray level
 - ▢ n_k : # of pixels with having gray level r_k
- ❑ Normalized histogram
 - ▢ $p(r_k) = n_k / n$
 - ▢ Σ all components of a normalized histogram = 1.
- ❑ Histogram manipulation for image enhancement.
- ❑ Histogram data is also useful in other applications.
 - ▢ Image compression
 - ▢ Segmentation
- ❑ Histograms are simple to calculate in software.

FOUR IMAGE TYPES

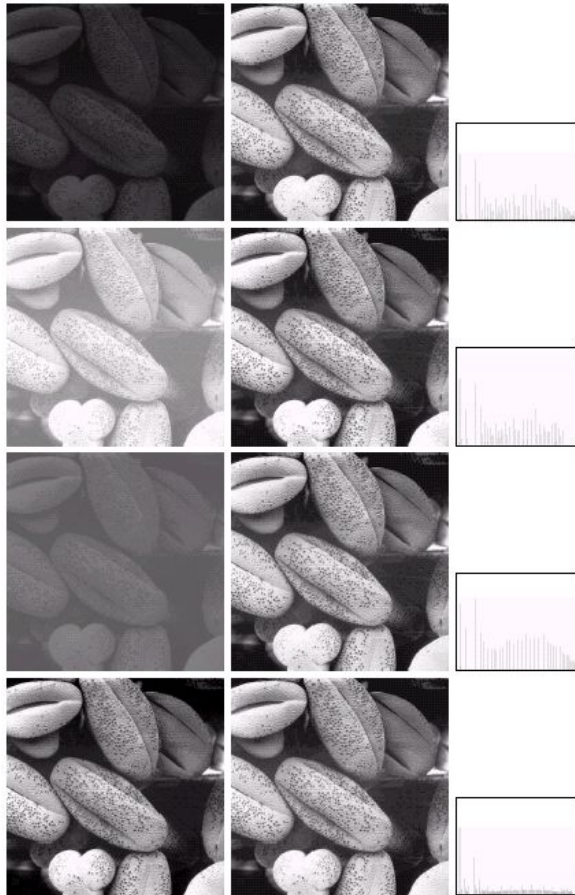


**Compare the four images
and their histograms.**

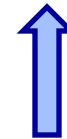
a b

FIGURE 3.15 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

HISTOGRAM EQUALIZATION

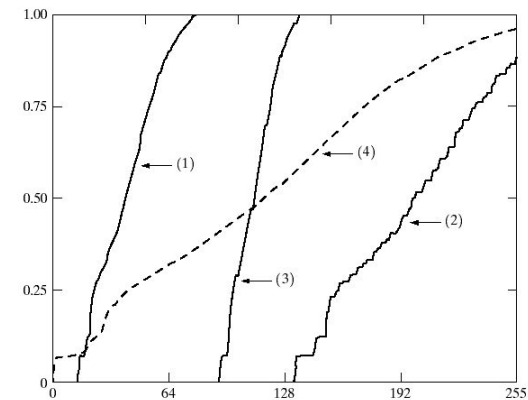


$$s_k = \sum_{j=0}^k \frac{n_j}{n}, k=0,1,2,...,L-1$$

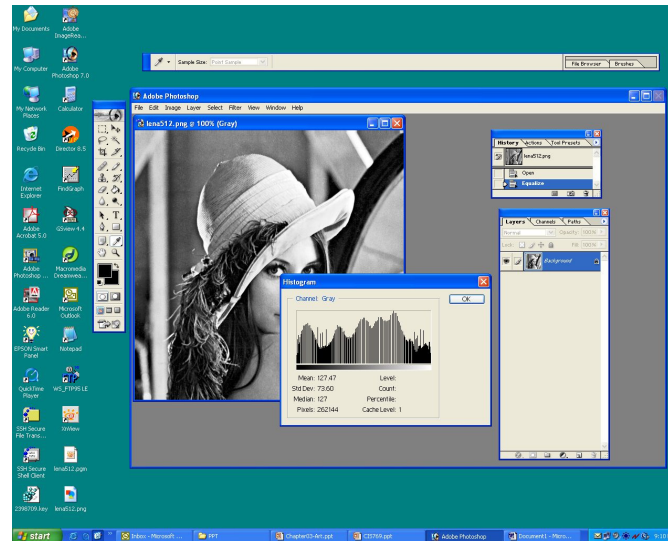
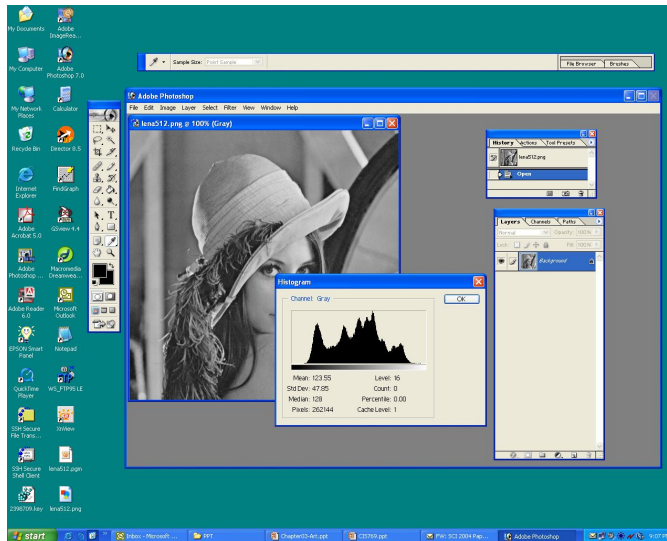


Has the general tendency of spreading the histogram of the input image.

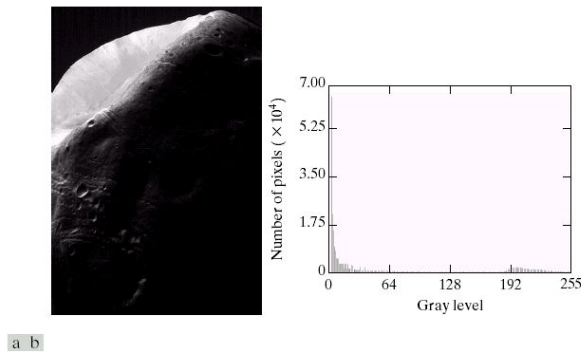
FIGURE 3.18
Transformation functions (1) through (4) were obtained from the histograms of the images in Fig.3.17(a), using Eq. (3.3-8).



AN EXAMPLE: LENA



HISTOGRAM MATCHING



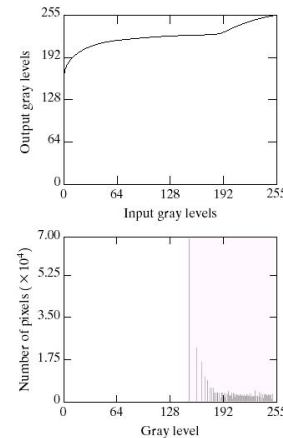
a b

FIGURE 3.20 (a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor*. (b) Histogram. (Original image courtesy of NASA.)

Histogram
equalization



Is the result
any good?



a b
c

FIGURE 3.21 (a) Transformation function for histogram equalization. (b) Histogram-equalized image (note the washed-out appearance). (c) Histogram of (b).

There are applications for which histogram equalization is not the best approach.

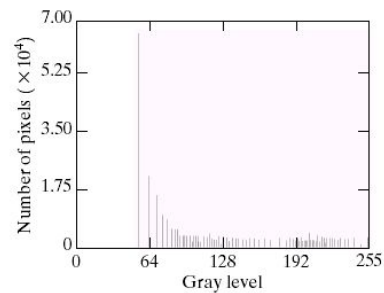
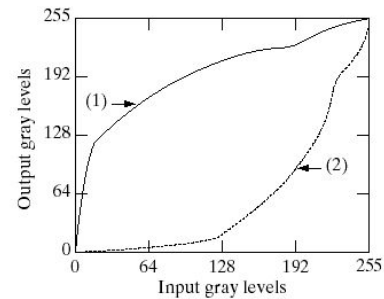
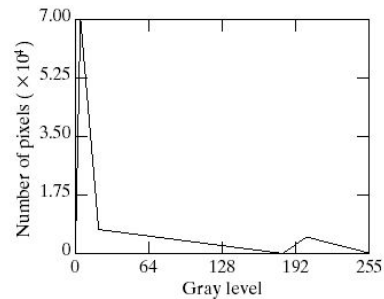
Sometimes it is useful to specify the shape of the histogram for the processed image.

Histogram matching: method to generate a processed image that has a specified histogram.

SPECIFIED HISTOGRAM

a c
b
d

FIGURE 3.22
(a) Specified histogram.
(b) Curve (1) is from Eq. (3.3-14), using the histogram in (a); curve (2) was obtained using the iterative procedure in Eq. (3.3-17).
(c) Enhanced image using mappings from curve (2).
(d) Histogram of (c).



Note that the low end of the histogram has shifted right toward the lighter region.

LOCAL ENHANCEMENT WITH HISTOGRAM PROCESSING

Histogram equalization
Histogram matching } ~~Global~~ methods for overall enhancement

Sometimes it is necessary to enhance details over small areas.

1. Define a neighborhood around a pixel
2. At each location
 - Compute the histogram
 - Obtain the transformation function
 - Use the function to map the gray level of the center pixel
3. Move the center to an adjacent pixel location.

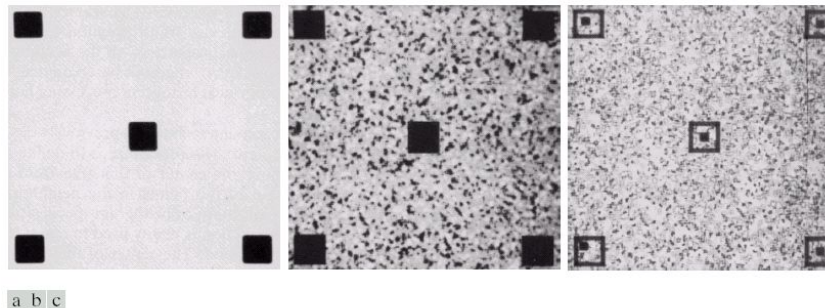


FIGURE 3.23 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a 7×7 neighborhood about each pixel.

HISTOGRAM STATISTICS FOR IMAGE ENHANCEMENT

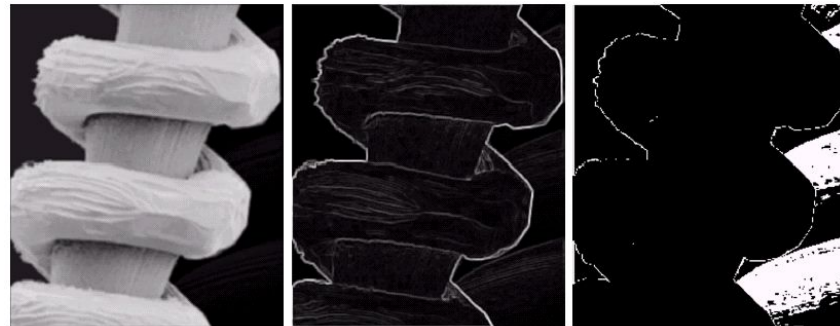
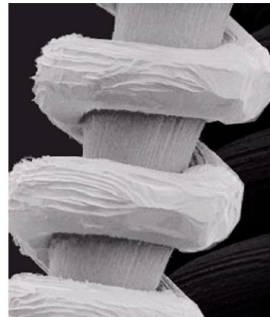
Local mean
Local variance

Can be used to develop simple yet powerful enhancement techniques!

$$g(x,y) = \begin{cases} Ef(x,y) & \text{if } m_{sxy} \leq k_0 M_G \text{ and } k_1 D_G \leq \sigma_{sxy} \leq k_2 D_G \\ f(x,y) & \text{otherwise} \end{cases}$$

Global mean Global SD

FIGURE 3.24 SEM image of a tungsten filament and support, magnified approximately 130×. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene).



a b c

FIGURE 3.25 (a) Image formed from all local means obtained from Fig. 3.24 using Eq. (3.3-21). (b) Image formed from all local standard deviations obtained from Fig. 3.24 using Eq. (3.3-22). (c) Image formed from all multiplication constants used to produce the enhanced image shown in Fig. 3.26.

$$E = 4.0, k_0 = 0.4, k_1 = 0.02, k_2 = 0.4$$

Selection of appropriate values requires experimentation.

ENHANCEMENT USING ARITHMETIC/LOGIC OPERATIONS

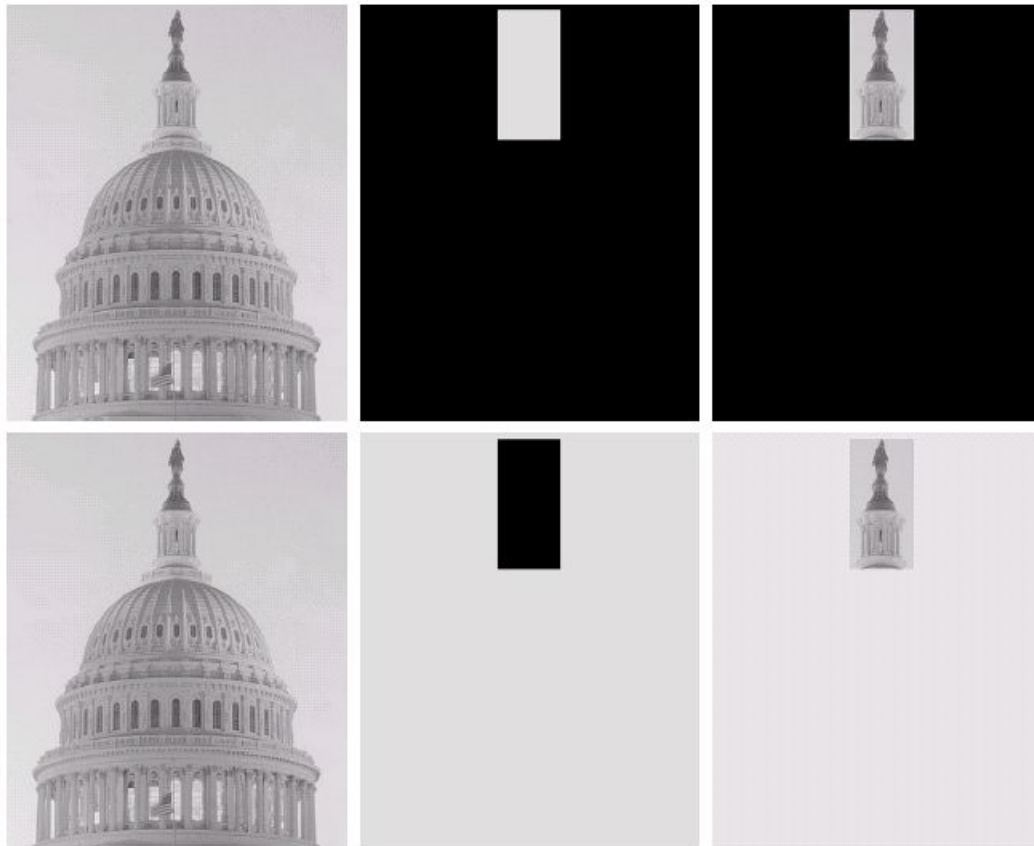
□ Arithmetic operations

- **Subtraction**
 - **Addition**
 - **Multiplication**: used as a masking operation
 - **Division**: multiplication of one image by the reciprocal of the other.
- } Most useful in image enhancement

□ Logical operations

- **AND**
 - **OR**
 - **NOT**
 - Frequently used in conjunction with morphological operations.
- } Used for masking
- } For these operations, gray scale pixel values are processed as strings of binary numbers.

AND/OR MASKS



a	b	c
d	e	f

FIGURE 3.27

(a) Original image. (b) AND image mask. (c) Result of the AND operation on images (a) and (b). (d) Original image. (e) OR image mask. (f) Result of operation OR on images (d) and (e).

IMAGE SUBTRACTION

$$g(x,y) = f(x,y) - h(x,y)$$

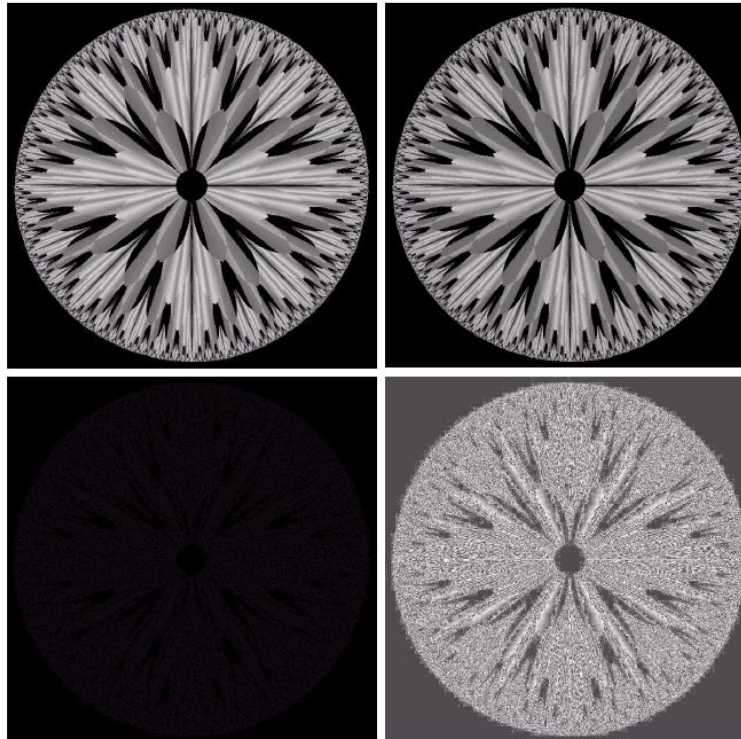
Two principal ways to **scale**:

1. Add 255 to every pixel and divide by 2.
2. Find the min difference and add its negative to all the pixel values. Then multiply each pixel by $255/\text{Max}$ (Max is the max pixel value in the modified difference image).

a b
c d

FIGURE 3.28

(a) Original fractal image.
(b) Result of setting the four lower-order bit planes to zero.
(c) Difference between (a) and (b).
(d) Histogram-equalized difference image.
(Original image courtesy of Ms. Melissa D. Binde, Swarthmore College, Swarthmore, PA).



← Contains more detail than the darker image.

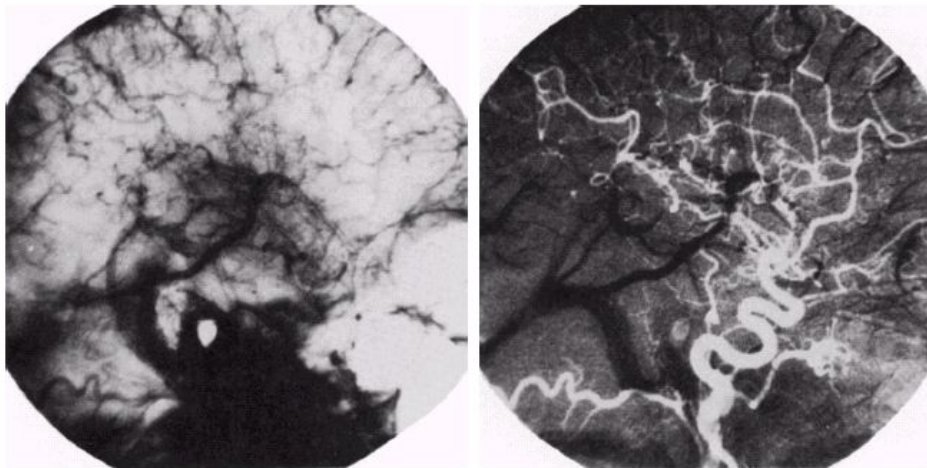
MASK MODE RADIOGRAPHY

The **mask** is an X-ray image of a region of a patient's body captured by an intensified TV camera.

A contrast medium is injected into the patient's blood stream.

A series of images are taken of the same region.

The net effect of subtracting the mask from each sample: areas that are different appear as enhanced detail in the output image.



a b

FIGURE 3.29
Enhancement by
image subtraction.
(a) Mask image.
(b) An image
(taken after
injection of a
contrast medium
into the
bloodstream) with
mask subtracted
out.

IMAGE AVERAGING

$$g(x,y) = f(x,y) + \eta(x,y)$$



At every pair of coordinates,
the noise is uncorrelated and
has zero average value.

Averaging K different **noisy** images $g_i(x,y)$, we get $f(x,y)$.

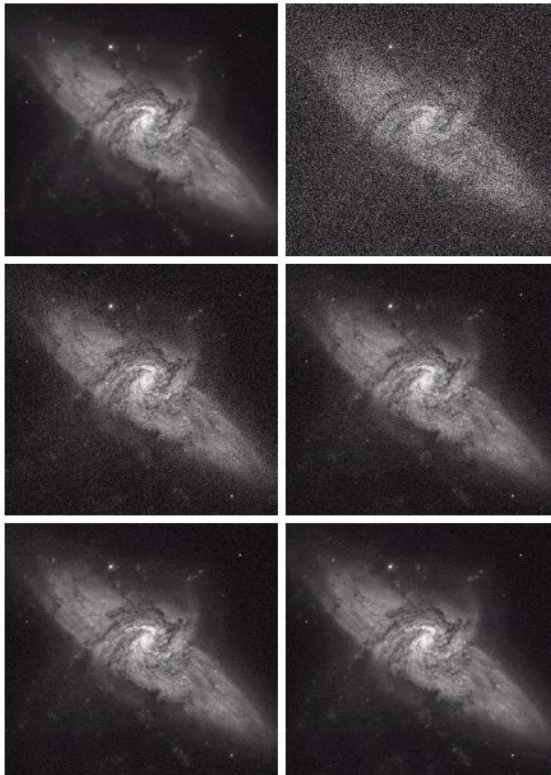
The **average** approaches $f(x,y)$ as the # of noisy images increases.

The images $g_i(x,y)$ should be **aligned** to avoid the introduction of artifacts.

As in the case of subtraction, **scaling** is needed.

- The sum of K 8bit images can range from 0 to $255K$.
- Image averaging may result in negative values.

GALAXY PAIR NGC 3314



a b
c d
e f

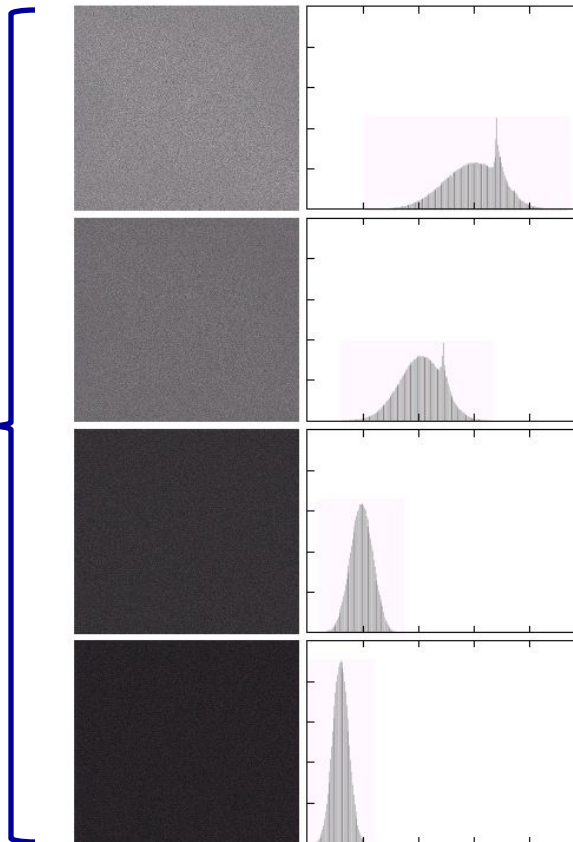
← Totally useless!

Averaging

FIGURE 3.30 (a) Image of Galaxy Pair NGC 3314. (b) Image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels. (c)–(f) Results of averaging $K = 8, 16, 64$, and 128 noisy images. (Original image courtesy of NASA.)

DIFFERENCE IMAGES AND THEIR HISTOGRAMS

Mean and **SD** of the difference images decrease as the value of K increases.



a b

FIGURE 3.31
(a) From top to bottom: Difference images between Fig. 3.30(a) and the four images in Figs. 3.30(c) through (f), respectively. (b) Corresponding histograms.

BASICS OF SPATIAL FILTERING

- ❑ The filter mask is moved from point to point in an image.
- ❑ At each point (x,y) , the response of the filter is calculated.
- ❑ Linear spatial filtering
 - ❑ 3x3 mask
 - ❑ $R = \omega(-1,-1)f(x-1,y-1) + \omega(-1,0)f(x-1,y) + \omega(0,0)f(x,y) + \dots$
 $+ \omega(1,0)f(x+1,y) + \omega(1,1)f(x+1,y+1)$
 - ❑ $\omega(0,0)$ coincides with $f(x,y)$ indicating that the mask is centered at $f(x,y)$.
- ❑ Our focus will be on masks of odd sizes.
- ❑ Image of size $M \times N$, filter mask of size $m \times n$.

$$g(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b \omega(s, t) f(x + s, y + t)$$

where $a = (m-1)/2$ and $b = (n-1)/2$

3x3 MASK

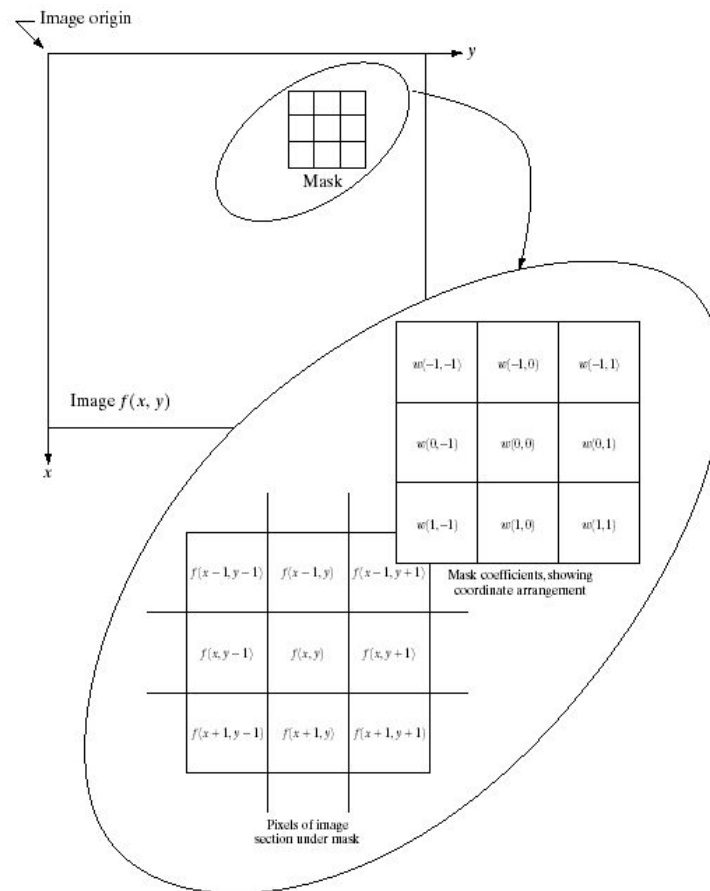
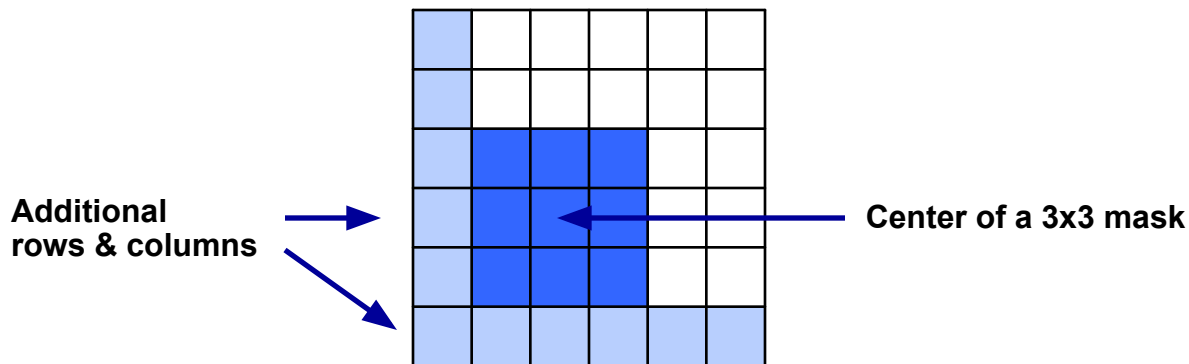


FIGURE 3.32 The mechanics of spatial filtering. The magnified drawing shows a 3×3 mask and the image section directly under it; the image section is shown displaced out from under the mask for ease of readability.

FILTERING AT THE BORDERS

- ❑ What happens when the center of the filter approaches the **border** of the image?
- ❑ Several alternatives:
 1. Limit the excursions of the center of the mask to be at a distance no less than $(n-1)/2$ pixels.
 2. Filter all pixels with only the section of the mask that is fully contained in the image.
 3. Pad the image by adding rows and columns of zeros (or other constant gray level).
 4. Pad the image by replicating rows and columns.



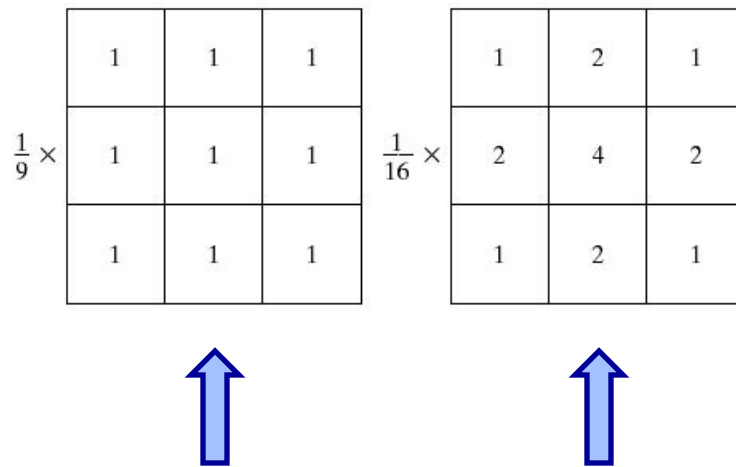
SMOOTHING SPATIAL FILTERS

- ❑ Used for **blurring** and **noise reduction**.
- ❑ Smoothing linear filters
 - ❑ Also called averaging filters or low pass filters.
 - ❑ The value of every pixel is replaced by the average of the gray levels in the neighborhood.
 - ❑ Standard average and weighted average.
 - ❑ Weighted averaging filter: Image of size $M \times N$, filter of size $m \times n$.

$$g(x, y) = \frac{\sum_{s=-a}^a \sum_{t=-b}^b \omega(s, t) f(x + s, y + t)}{\sum_{s=-a}^a \sum_{t=-b}^b \omega(s, t)}$$

- ❑ Order-statistics filters
 - ❑ Nonlinear spatial filters
 - ❑ Best-known example: Median filter

TWO 3X3 SMOOTHING FILTERS



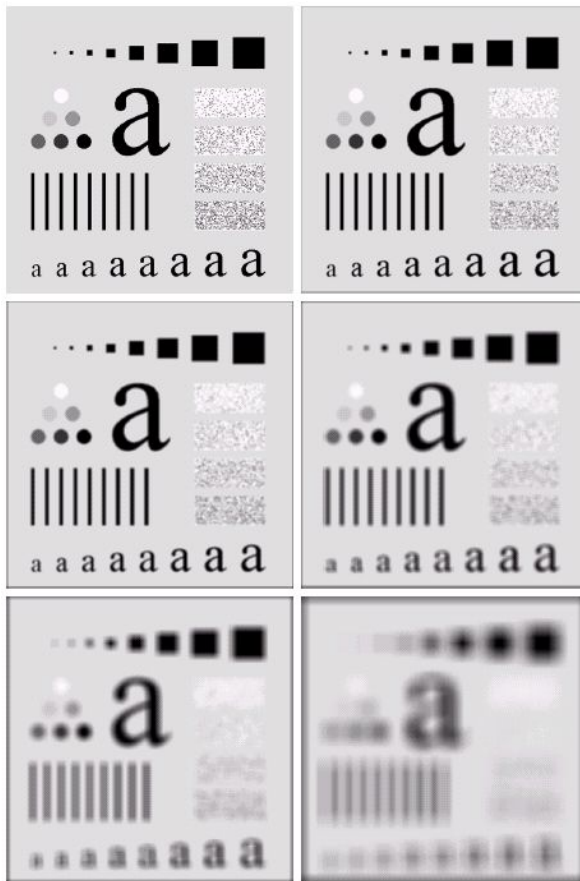
A box filter

The basic strategy is
to reduce blurring.

a b

FIGURE 3.34 Two 3×3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to the sum of the values of its coefficients, as is required to compute an average.

FIVE DIFFERENT FILTER SIZES

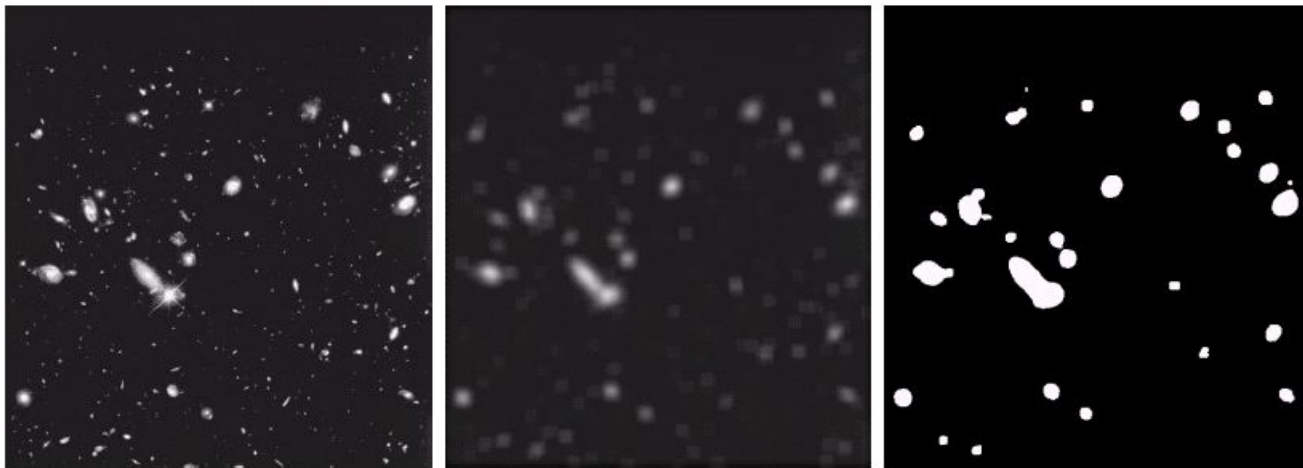


a b
c d
e f

FIGURE 3.35 (a) Original image, of size 500×500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $n = 3, 5, 9, 15$, and 35 , respectively. The black squares at the top are of sizes $3, 5, 9, 15, 25, 35, 45$, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their gray levels range from 0% to 100% black in increments of 20% . The background of the image is 10% black. The noisy rectangles are of size 50×120 pixels.

Pronounced **black border** is the result of padding the border with 0's, and the trimming off the padded area.

BLURRING AND THRESHOLDING



a b c

FIGURE 3.36 (a) Image from the Hubble Space Telescope. (b) Image processed by a 15×15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

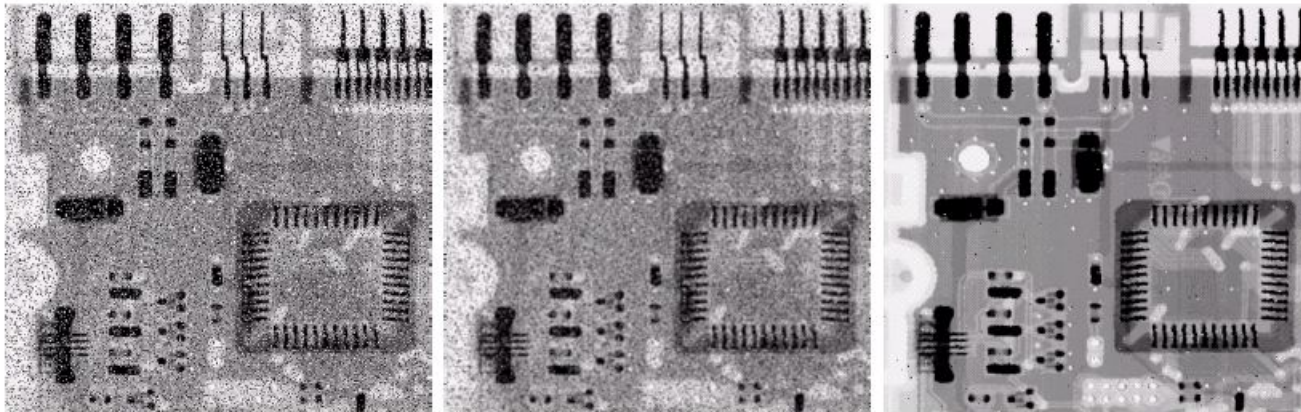
↑
**THRESHOLD = 25% of highest
intensity in the blurred image**

ORDER-STATISTICS FILTERS

- ❑ Response of the filter is based on ordering (ranking) the pixels.
- ❑ The value of the center pixel is replaced by the value determined by the ordering result.
- ❑ **Median** filters
 - ❑ For certain types of noise, they provide excellent noise reduction with less blurring than linear filters of similar size (very effective in the presence of salt-and-pepper noise).
 - ❑ The median, ξ , of a set of values: half of the values $\leq \xi$, and half $\geq \xi$.
 - ❑ 3x3 neighborhood: median is the 5th largest value.
 - ❑ 5x5 neighborhood: median is the 13th largest value.
 - ❑ Principal function is to force points with distinct gray levels to be more like their neighbors.
- ❑ **Max** filter: 100th percentile
- ❑ **Min** filter: 0th percentile

EFFECT OF AVERAGING AND MEDIAN FILTERS

Which one has removed the salt-and-pepper noise??



a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

SHARPENING SPATIAL FILTERS

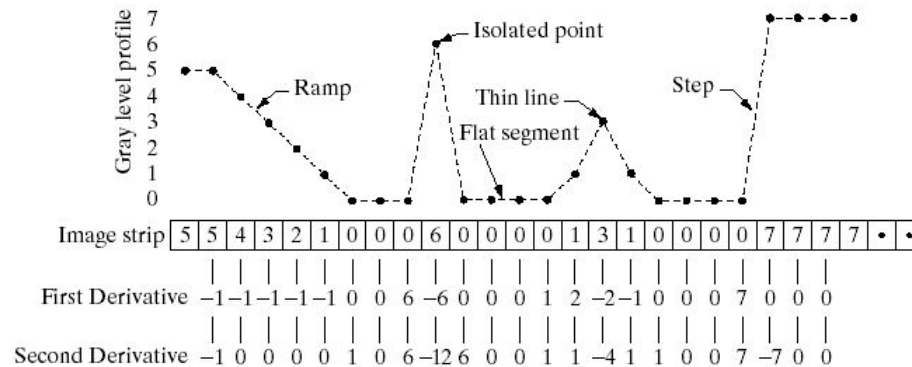
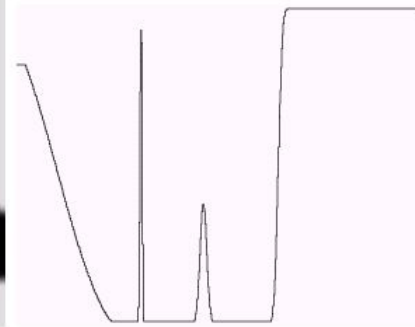
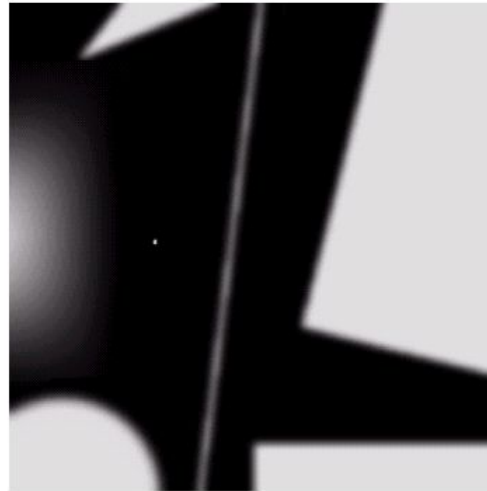
- ❑ Used for **highlighting fine detail** or **enhancing detail** that has been blurred.
- ❑ Averaging is analogous to **integration**.
- ❑ Sharpening is analogous to **differentiation**.
- ❑ **First derivatives**
 - ▢ $f'(x) = f(x + 1) - f(x)$
 - ▢ Must be zero in flat areas
 - ▢ Must be non-zero at the onset of a gray-level step or ramp
 - ▢ Must be nonzero along ramps
- ❑ **Second derivatives**
 - ▢ $f''(x) = f(x + 1) + f(x - 1) - 2f(x)$
 - ▢ Must be zero in flat areas
 - ▢ Must be non-zero at the onset and end of a gray-level step or ramp
 - ▢ Must be zero along ramps of constant slope

1ST AND 2ND DERIVATIVES

a b
c

FIGURE 3.38

(a) A simple image. (b) 1-D horizontal gray-level profile along the center of the image and including the isolated noise point. (c) Simplified profile (the points are joined by dashed lines to simplify interpretation).



2ND DERIVATIVES OF ENHANCEMENT – THE LAPLACIAN

$$f'_x(x,y) = f(x+1,y) + f(x-1,y) - 2f(x,y)$$

$$f'_y(x,y) = f(x,y+1) + f(x,y-1) - 2f(x,y)$$

$$\nabla^2 f = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) & \text{if the center coefficient is “-”} \\ f(x,y) + \nabla^2 f(x,y) & \text{if the center coefficient is “+”} \end{cases}$$

IMPLEMENTATIONS OF THE LAPLACIAN

0	1	0	1	1	1
1	-4	1	1	-8	1
0	1	0	1	1	1
0	-1	0	-1	-1	-1
-1	4	-1	-1	8	-1
0	-1	0	-1	-1	-1

a	b
c	d

FIGURE 3.39

(a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4).

(b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.

SHARPENED NORTH POLE OF THE MOON

a b
c d

FIGURE 3.40

(a) Image of the North Pole of the moon.
(b) Laplacian-filtered image.
(c) Laplacian image scaled for display purposes.
(d) Image enhanced by using Eq. (3.7-5).
(Original image courtesy of NASA.)



$g(x,y)$ is usually implemented with one pass of a single mask.

$$g(x,y) = f(x,y) - \nabla^2 f(x,y)$$



0	-1	0
-1	5	-1
0	-1	0

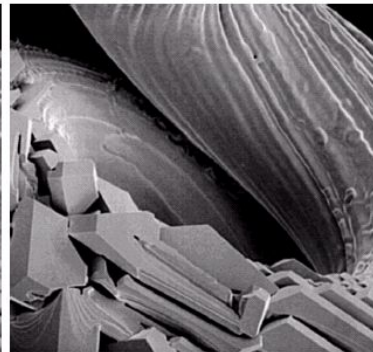
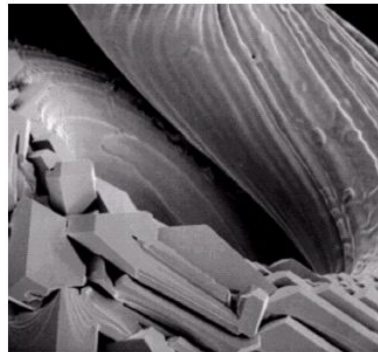
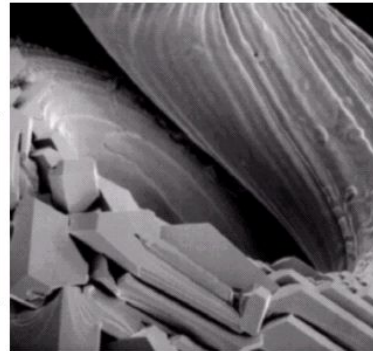


Small details are enhanced
And the background tonality is perfectly preserved.

TWO LAPLACIAN MASKS

0	-1	0
-1	5	-1
0	-1	0

-1	-1	-1
-1	9	-1
-1	-1	-1




a b c
d e

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

UNSHARP MASKING & HIGH-BOOST FILTERING

Unsharp masking: $f_s(x, y) = f(x, y) - \bar{f}(x, y)$



 Blurred version of f

High-boost filtering: $f_{hb}(x, y) = Af(x, y) - \bar{f}(x, y)$

0	-1	0	-1	-1	-1
-1	$A + 4$	-1	-1	$A + 8$	-1
0	-1	0	-1	-1	-1

a b

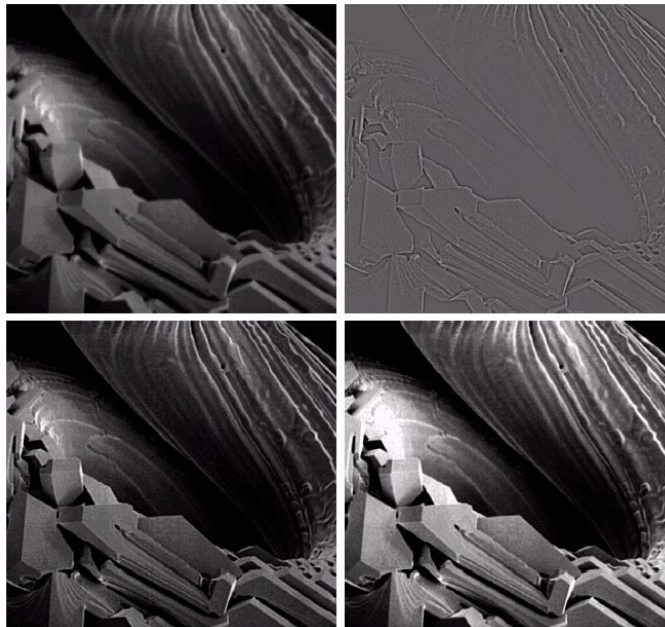
FIGURE 3.42 The high-boost filtering technique can be implemented with either one of these masks, with $A \geq 1$.

AN APPLICATION OF BOOST FILTERING

a b
c d

FIGURE 3.43

(a) Same as Fig. 3.41(c), but darker.
(b) Laplacian of (a) computed with the mask in Fig. 3.42(b) using $A = 0$.
(c) Laplacian enhanced image using the mask in Fig. 3.42(b) with $A = 1$.
(d) Same as (c), but using $A = 1.7$.



-1	-1	-1
-1	$A+8$	-1
-1	-1	-1

Laplacian with $A = 0$

1ST DERIVATIVES OF ENHANCEMENT – THE GRADIENT

a
b c
d e

FIGURE 3.44

A 3×3 region of an image (the z 's are gray-level values) and masks used to compute the gradient at point labeled z_5 . All masks coefficients sum to zero, as expected of a derivative operator.

The weight value of 2 is used to achieve some smoothing by giving more importance to the center point.

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

-1	0	0	-1
0	1	1	0

-1	-2	-1	-1	0	1
0	0	0	-2	0	2
1	2	1	-1	0	1

$$\nabla f \approx |G_x| + |G_y|$$



$$\nabla f \approx |z_9 - z_5| + |z_8 - z_6|$$

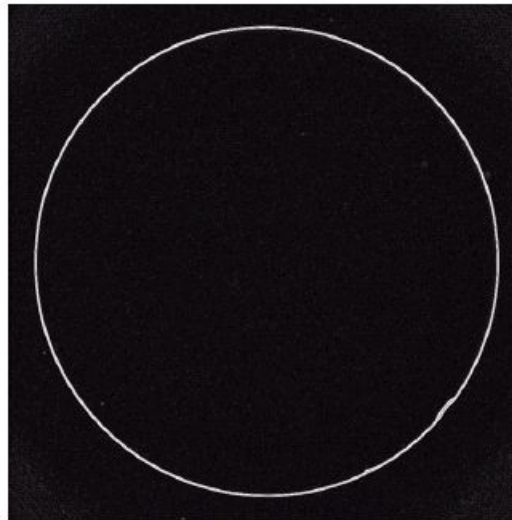
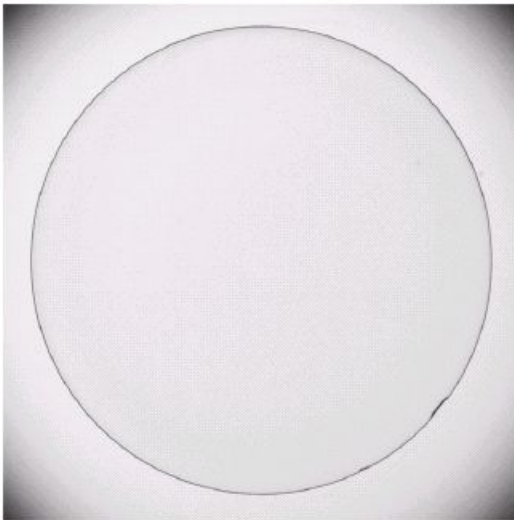


Sobel operators

$$\begin{aligned} \nabla f \approx & |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| \\ & + |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)| \end{aligned}$$

APPLICATION OF SOBEL OPERATORS

Gradient is frequently used in industrial inspection.



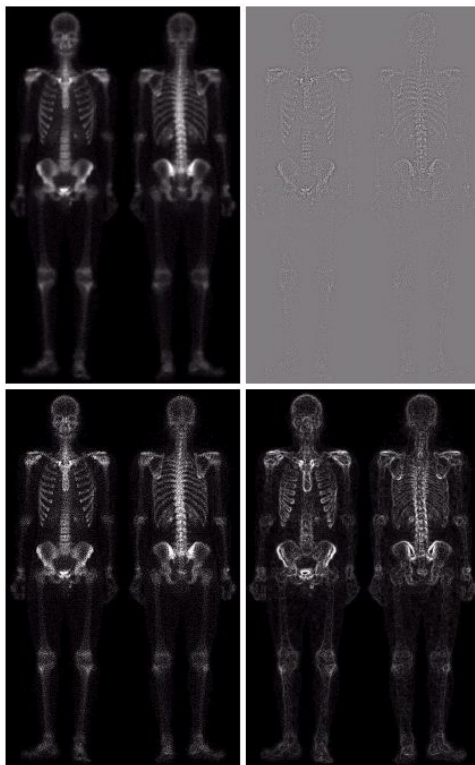
a b

FIGURE 3.45
Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock).
(b) Sobel gradient.
(Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)



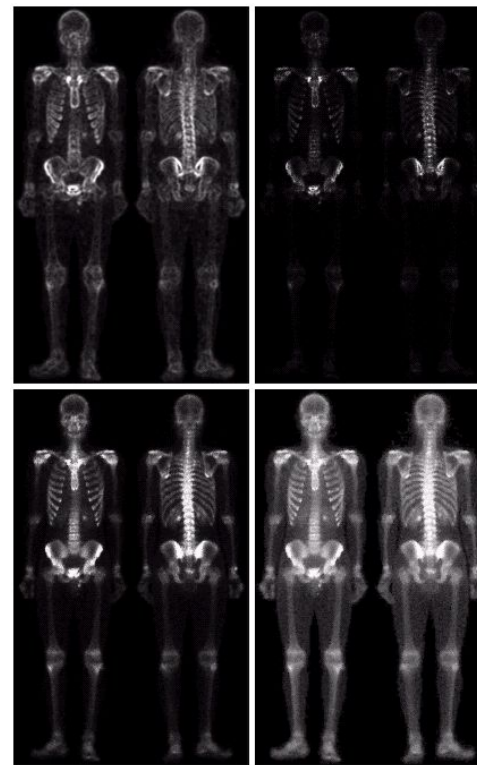
Constant or slowly varying shades of gray have been eliminated.

COMBINING SPATIAL ENHANCEMENT METHODS



a b
c d

FIGURE 3.46
(a) Image of whole body bone scan.
(b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel of (a).



e f
g h

FIGURE 3.46
(Continued)
(e) Sobel image smoothed with a 5×5 averaging filter. (f) Mask image formed by the product of (c) and (e).
(g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)